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MAGNETIC FIELDS OF FORCE

MAGNETIC FIELDS OF FORCE

AN EXPOSITION OF THE PHENOMENA OF
MAGNETISM, ELECTRO-MAGNETISM, AND INDUCTION
BASED ON THE CONCEPTION OF

LINES OF FORCE

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PART I

WITH 93 ILLUSTRATIONS

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AUTHOR'S PREFACE

THE epoch-making researches of Heinrich Hertz have now given definite confirmation to those 'modern' conceptions of the nature of magnetic, electric and optical phenomena which originated with Faraday, and were further developed by Maxwell and by Helmholtz. Those conceptions have at length become triumphant throughout the entire subject, though they had long to contend with the older theories for supremacy.

In the present treatise I have tried the experiment of using the conception of lines of force in the systematic exposition of the phenomena concerned, and have attempted to build up from first principles the laws of Magnetism and of Electricity, in accordance with the latest developments of theoretical knowledge. Many attempts of this kind have already been made, but they are for the most part in the form of special treatises which deal more or less exhaustively with some limited portion of the subject, or which only sketch cursorily the deductive processes involved. So far there has been no convenient text-book developing these new conceptions throughout the whole subject in an easily intelligible manner, so that they may be considered as coming within the scope of a course of lectures on Experimental Physics. The present treatise was, indeed, the outcome of such lectures, wherein I became convinced of the high didactic value which is inherent in the new conceptions. Especial stress was laid on three fundamental conceptions (as indeed is done in most text-books): the conception of lines of force, the conception of energy in a magnetic or electro-magnetic field, and that of the symmetry of a medium which is the seat of magnetic force.

It is characteristic of the modern development of electrical and magnetic theory that the seat of the phenomena is to be found in the region or 'field' surrounding the attracting bodies. We no longer suppose magnetic attractions to be due to fluids concentrated at certain points, but to pressures and tensions in the field. When an electrical current is flowing, the seat of the phenomenon is not the conductor merely; the most important part of the whole phenomenon takes place in the field of magnetic force surrounding the conductor. It is accordingly to these fields, and the forces continuously propagated through them from point to point, that the highest interest attaches in the new theory. Their form and structure we shall study by means of the lines of force.

The introduction and logical application of the conception of lines of force enable us to treat the entire subject more connectedly and with more systematic completeness. The separate groups of phenomena are no longer unconnected, as in the historical development of the science, but all the facts form a connected chain, and are logically and inevitably bound up with one another. Moreover, the separate laws are made much more readily intelligible, since they can be immediately deduced from diagrams of lines of force. Finally, it is of importance, in elementary instruction, to use those methods of exposition which form the basis of the prevailing theory, and which play the most important part in the working out of elaborate technical questions, were it only that by this means continuity would be secured in the studies of those who desired to pursue the subject further, in the one direction or the other.

The value of considerations concerning energy are still disputed by some; but it cannot be denied that the conception of energy plays a very important part in electrodynamics. It can be shown that the essential difference between the two theories as to the nature of electrical and magnetic fields is to be found in the expression for the energy in each separate volume-element of the region concerned. An exposition of the principles of the localisation and migration of energy has been given here as being in the highest degree fruitful and suggestive; and the most important applications of the subject—the production of currents by induction, the working of motors, and the exertion of force—depend upon the transformation of mechanical forms of energy into energy of the electro-magnetic field, and conversely. Since the consideration of these energy-transformations plays so fundamental a part in this book, I might have chosen for its title 'Magnetic Fields of Energy.' In naming it, however, I have confined myself to one factor of the energy, the force; because the distribution of energy can be mapped out by means of the Faraday-Maxwell lines of force or lines of induction alone.

The symmetrical properties of a field which is the seat of magnetic actions decide upon the tenability or untenability of a dynamical hypothesis or a diagrammatic representation designed to elucidate the nature of magnetic stress.

This first part of the book is intended to serve rather the purpose of an introduction; the new method—important in the later developments of the subject—is illustrated and explained by applying it to examples which are simple and for the most part quite elementary. Elaborate mathematical calculations are entirely avoided, experimental data being taken as the basis of the investigations. Departing somewhat from the usual practice, the fundamental conception of a magnetic field and lines of force is introduced in connection with natural magnets, and not with magnets of artificial forms. In explaining the laws of the electric current, the greatest prominence is given to the properties of the field, while similarly in the chapter on electrodynamics the first place is given to those things which are the most important in their applications, so that, for example, the first current-producing apparatus described is the dynamo-machine.

The second part will deal with the phenomena of induction, and, as before, qualitative relations will be dealt with apart from quantitative. We shall then have to consider the phenomena of self-induction, single-phase and multiphase periodic currents, and rotatory magnetic fields. Then follows an exposition, based on Helmholtz's 'Theory of Cycles,' of the transformation into heat of the energy of the field (resistance), and the accumulation of energy in unclosed circuits (condensers, capacity). The phenomena of electrical oscillations are treated together with those of optics in a condensed account of electro-optics, including a discussion of the conditions necessary for the generation of longitudinal disturbances, and an account of the luminous phenomena in rarefied gases, as well as the laws of propagation of the effects of electrical discharges, kathode-rays and the Röntgen-rays, from the standpoint of Maxwell's theory. Finally, there are the researches which Heinrich Hertz has left us as a testament in his posthumous work, 'Prinzipien der Mechanik,' and which will be treated at length in the last section of the book. They are a further development of the conclusions of Helmholtz, and are concerned with the question of the essential nature of electricity, by which we are to understand the reduction of all the various phenomena of this agent to a single intelligible and connected system. The subject is only treated tentatively and suggestively in the second part of the 'Prinzipien,' the true solution of the question appearing so far to be insufficiently known. Hertz commences with certain properties of the expression for the energy in what are called polycyclic systems, and in this I desire to follow him, attempting, however, to deal in their individual aspects with his latest additions to our knowledge of electrical phenomena, apart from the general system of his mechanics.

Many of the models and pieces of apparatus described in this book are new, and such of them as are very simple have been used in my lectures at Erlangen, Leipzig and Kiel to exemplify conceptions and laws fundamental in the modern theory.

It will further be remarked, and possibly with disapproval, that in some places the treatise has the character of a book on laboratory-practice, the typical experiments being described in such a manner that the reader can perform them for himself. This has been done from the conviction that in this way the true significance of the experiments would be the more fully and readily grasped.

I am indebted to my assistant, Herr HANS BOAS, for the preparation of the diagrams of lines of force, to Herr LUDWIG RELLSTAB, student in the University of Kiel, for assistance in reading the proofs of the book.

My thanks are especially due to the publishers, who have spared no pains to make the reproduction of the diagrams and the general printing of the book as perfect as possible.

HERMANN EBERT.

KIEL: *April* 1896.

TRANSLATOR'S NOTE

THE Author's Preface sufficiently explains the aim and scope of his work, and little remains for me to say on that head. I have endeavoured, as nearly as might be, to make this volume a faithful translation of the German original, the changes introduced being few, and for the most part very slight. They are in all cases distinguished by the use of square brackets. Wherever it seemed necessary to make any important change in the text, the author was first consulted, and his sanction obtained; the chief instance being in § 119, where the demonstration differs considerably from that in the original volume.

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SECTION I

THE PHENOMENA OF MAGNETISM

CHAPTER I

THE MAGNET AND THE MAGNETIC FIELD

A.—The simple natural magnet

1. **The lodestone.**—It was known to the ancients that certain ores of iron possess the property of attracting and holding to them, in a peculiar way, small particles of iron. Most notable in this respect was an ore from a mine in the province of Magnesia, in Asia Minor, so that the term 'magnetic stone' (ἡ Μαγνῆτις λίθος) came to have a special significance, the name 'magnetic' being applied to the entire group of phenomena with which we are now concerned.

The stone itself is shortly called a magnet, or, to distinguish it from those which are artificially produced, a 'natural magnet.'

The lodestone, or magnetic iron ore, is a combination of equivalent parts of ferrous oxide and ferric oxide, and has the formula $Fe\,O, Fe_2\,O_3$. It is found for the most part in dense homogeneous pieces of a lustrous black colour, and when crystallised has the form of the regular system. Next to the lodestone in magnetic properties is the magnetic sulphide of iron ($Fe\,S$, $Fe_2\,S_3$). The same properties are found to exist to a much smaller extent in the ores of nickel and cobalt.

2. **The magnetic property.**—*Experiment* 1.—Let a piece of lodestone be dipped in iron filings: tufts and strings of filings remain hanging to the lodestone in many places, though not in all, more especially at projecting corners and edges.

If any other body, such as a finger, be dipped into the filings, it may be that some few particles will remain adhering to it, but the adhesion is evidently of an entirely different kind. The comparison shows that the lodestone exerts some special influence on the iron filings.

If the experiment be repeated on substances other than iron filings, the lodestone being dipped, for example, into particles of copper or brass, or into silica ($Si\,O_2$) in the form of sand, we shall only notice that kind of cohesion which exists between all bodies in contact, and not any special action of the lodestone on the particles.

It is not to be concluded from this that the bodies in question are never susceptible to magnetic influence. We shall find later, when we have learnt how to produce much stronger magnetic effects, that all substances, even gases, respond in some measure to magnetic influences. Our experiment only shows, then, that iron responds in a far more notable degree. We shall accordingly make use of iron for the closer investigation of magnetic phenomena. We speak of it as 'strongly magnetic' in distinction from 'feebly magnetic' bodies. There are no unmagnetic bodies.

3. **Conception of a magnet.**—Any body which exerts the same influences as the lodestone we shall call a 'magnet.'

Accordingly we should speak of the so-called 'galvanic current' as a magnet, because in treating of it we start from a consideration of its magnetic influence, which we make directly evident by means of iron filings, just as in the present section we do with all ordinary magnets.

4. **Closer investigation of the nature of magnetic effects.** *Experiment* 2.—On examining a piece of lodestone which has been dipped in iron filings, the adherent tufts appear not only to be subject to the influence of gravity, but to be themselves the seat of some special kind of influence. Between the separate particles some modifying cause is evidently at work; they possess a certain rigidity. If we try to separate them from the lodestone they become drawn out into chains, the filings adhering one to another in succession.

This is seen especially clearly when the filings are supported from below, so that they are only in part dependent from the magnet.

Experiment 3.—Scatter some iron filings over a piece of white paper, and over the sheet draw a corner of the lodestone which was found in experiment 1 to be especially active: a string of filings forms and grows in length, the particles already attached acquiring the property of the lodestone, and attracting others.

The magnetic influence is thus taken up by the chains of particles, and transmitted from one particle to another.

The effect is greatest near to the lodestone, and diminishes at greater distances; when the chain is too long it breaks off, the successively diminishing magnetic forces being finally insufficient to overcome the frictional forces conditioned by the weights of the separate particles.

5. **Diagrams of lines of force.**—The method of procedure so far employed, of dipping a lodestone into iron filings, and examining the manner in which the influence is taken up and transmitted from particle to particle, is, however, imperfect. We must further take care that, in the building up of the chains of influence which we wish to study, the effect of gravity shall be as far as possible eliminated. This result can be most easily attained by scattering the filings over a horizontal supporting surface, which passes through a region of magnetic influence.

Experiment 4.—By means of a sieve, let iron filings be scattered from a greater height, so that they may be distributed as uniformly as possible over the horizontal sheet of paper. Next let the lodestone, freed from any filings that may already be adhering to it, be laid upon the paper, which is then tapped in various places. The filings arrange themselves in regularly curved lines (fig. 1); these are the so-called *Faraday's lines of force.*

These iron-filing pictures were known before Faraday's time; but their true significance was not appreciated. Faraday made them the starting point of a series of important and fundamental conceptions, and accordingly we name them after him.

These lines of force show very clearly how the influence exerted by the lodestone and transmitted from filing to filing is in a certain sense 'conducted.' It should be observed that, even at points somewhat distant from the lodestone, the filings take up a definite direction, corresponding to the course of the lines of force which pass through their positions.

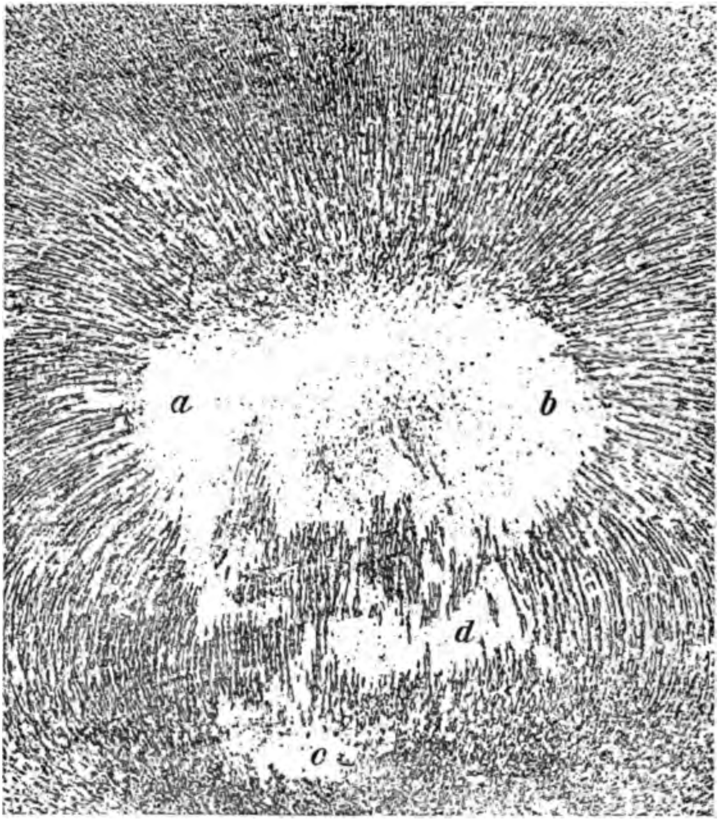

Fig. 1

6. **Practical hints on the production of pictures of magnetic lines of force.**—As we shall often avail ourselves in the following pages of the very important aid of these line-of-

force diagrams, it may be as well in this place to make some remarks upon their production.

1. Very striking, but somewhat coarse pictures are obtained by using fine iron filings; they are especially suited for demonstration before a large audience.

That the filings may be as black as possible against the white paper, they should be once heated to redness before being used. Part of the iron will in this way be changed into ferro-ferric oxide, which (§ 1) is also susceptible to magnetic influences; at the same time any oil or grease adhering to the filings will be burnt off. The sieve used should be free from iron; for example it may be of porcelain.

2. If the finer features of the diagram are to be studied, *ferrum pulveratum* should be employed, or the finest iron powder, *ferrum redactum*. A small linen bag should be filled with this and shaken, when a fine grey powder will be distributed over the paper.

If kept for long the finely powdered iron tends to absorb moisture. It becomes caked together and produces disturbing spots on the diagram. To avoid this it should be heated red hot (for example on a copper plate before the blowpipe). When ground in a mortar it then yields a very fine black powder, which can be quite easily sifted through linen. In no case should it be strewn too thickly over the paper.

The best paper to use is smooth white cartridge paper. If the magnetic forces are very strong the filings rush over the surface of the paper, and stand up in high projecting tufts. In this case it is better to use rough writing-paper, or in some instances even coarse-grained drawing-paper.

For purposes of projection, line-of-force diagrams on glass plates may be used, though they are not to be recommended. They must be brought in a horizontal position into the beam proceeding from the lantern.[1] If only the picture itself is to be shown, it must be fixed by one of the methods described in the following paragraph.

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[1] The beam, which ordinarily leaves the lantern horizontally, must thus be made to travel vertically through a short part of its course, which is best effected by means of two large totally-reflecting prisms.

To make the diagrams sharp and clear, the sheet of paper should be tapped with a little hammer (made by sticking a cork on to a piece of fish-bone), by which means the particles of iron will be for an instant raised clear of the paper.

If it is desired to tap the paper at particular points, or to hold other points at rest to keep them from being further shaken, a pointed glass rod will be found useful. By tapping in this way the diagram has to be, as it were, hammered into shape.

Other methods which serve to give an idea of the special distribution of magnetic lines of force will be described in connection with the more powerful magnetic forces which are essential to their employment.

7. Fixing the line-of-force diagrams.—For rendering permanent the figures obtained under the influence of a magnet several methods may be employed.

1. *Fixing by means of a spray of shellac.*—As soon as a clear diagram of lines of force has been obtained, let the paper be sprayed over with a thin solution of white shellac or mastic in alcohol; as soon as the solution has dried, the particles of iron will be held firmly in position. No large drops should be allowed to issue from the spray-diffuser, nor should the spray be turned so as to play directly upon the paper.

2. *Transference to gummed paper.*—Upon the diagram lay a sheet of paper, thinly and uniformly coated with starch-paste or gum-arabic, and press with a handkerchief or with the ball of the hand; then remove the gummed sheet (PFAUNDLER). The gum should not be too thin, or it will adhere to the lower sheet. It is better, therefore, to allow the surface to dry partially before applying it. If the transferred diagram is intended for purposes of projection, a glass plate must be used, upon which the gum is spread and allowed to become nearly dry; after the transfer has been effected, streaks and bubbles may be easily removed by means of a camel's hair pencil. Figs 1, 3, 5, 12, 13.

3. *Fixing by softening the underlay.*—The sheets of paper or glass plates on which the diagrams are to be prepared are coated with a solution of shellac and allowed to dry; the so-called negative varnish, which is commonly used for coating gelatine plates, will be found very suitable for the purpose. When the line of-

force diagrams have been obtained, a little of the same solution, or even of alcohol, is sprinkled on the plate, and the particles of iron are thus held fast in position. Figs. 10, 11, 15 are photographic reductions of diagrams fixed in this way. On gently warming, the layer of shellac becomes sufficiently transparent for the diagram to be copied photographically by transmitted light (daylight).

Another way is to cover a sheet of paper or glass with a thin layer of paraffin-wax, and after forming the line-of-force diagram to fix by warming.

4. *Photographic fixing.*—The following method gives results richest in detail, and at the same time best suited for purposes of projection. In the dark-room the gelatine film of a bromide of silver dry-plate is sprinkled over as evenly as possible with the finest iron dust; then the body exerting magnetic influence is laid upon the plate, or placed underneath it, and the edges of the plate are tapped with a glass rod. On lighting the lamp of the dark-room, or by means of the light which comes through the red glass of the window, it is easy to ascertain whether the iron dust has separated into fine and regular lines. The magnet is then removed from the dark-room and illuminated for a short time along with the iron-sprinkled negative. (The light of a lucifer match at a distance of a metre answers perfectly with ordinary plates.) After this exposure the iron dust is blown off the plate, which is then brushed over with a soft dry brush, and developed and fixed in the usual way. The black chains of opaque iron particles appear bright against a dark ground. From the negatives, positives can be printed on paper or on glass, the glass positives giving very satisfactory pictures when projected by means of the lantern. They may be finally 'intensified' by means of mercuric chloride, so as to increase the contrast. Negatives which are to serve for purposes of projection should be developed so as to have less density than those intended for printing out. Figs. 2, 4, 6, 16, were obtained in this way; while figs. 73, 75, 76, 77, 78 of Section II. were reproduced from photographs, directly taken from figures on paper. In the latter case the camera was supported at the proper height above the paper, with the objective pointing downwards.

Finally, the ordinary method of exposure may be advantageously used, especially for large figures, and with coarser particles of iron dust. The figures are formed on sensitised paper, and after exposure the picture is fixed in the usual way.

8. **The line-of-force diagram of a natural magnet.**—Fig. 1, which has already been mentioned in § 5, must now be considered more closely.

a. At corners and edges, where experiments 1 and 2 showed the magnetic force to be strongest, the particles of iron adhere closely to the lodestone, having left more distant points to rush to these places. When the magnet is taken from the diagram, these particles are removed along with it, and this explains the light patches *a*, *b*, *c*, *d*.

β. From these places of greater activity lines of force are seen to proceed. Those proceeding (for example) from the position *a* do not pass in straight lines from *a* to *c* following the shortest course, but are bowed out into arcs, more or less strongly curved, and concentrate again towards *c*, where they terminate. The same is true of *b* and *d*.

γ. The lines of force are crowded together in the neighbourhood of *a* and *b*, and on their way from *a* to *b* are spread further apart. They are therefore densest in the neighbourhood of these places, and are more thinly scattered as we proceed to greater distances; that is, they crowd most closely together in those places where experiment 1 showed the magnetic influence to be strongest.

Accordingly the line-of-force diagrams furnish an indication of the direction and magnitude of the magnetic force.

9. **Polarity and pole.**—From a certain contrariety between the properties of different points at the surface of the magnet, it may be concluded that the influences which proceed from some places re-enter the magnet at other places. This does not apply to the simple attachment of iron filings to the magnet, but to the outgoing and returning of lines of force.

Both kinds of places are called poles (from πόλος = axis, πέλω = I turn), like the opposite ends of the axis of rotation of our planet; and the contrariety of properties which they exhibit is called 'polarity.'

We shall refer to all those parts of a magnet where the issuing (or re-entering) lines of force are most numerous as polar regions, or, more shortly, as poles.

We shall afterwards have to consider more closely the analogous property of the polar regions of our earth; from one of these, magnetic lines of force proceed outwards into space, while at the other the lines pass back into the interior of the earth.

10. Localisation of the poles of a natural magnet.—*Experiment* 4′.—Take several sheets of paper, and in each cut an opening of such shape and size that some particular cross-section of the magnet will just fix within it. Disposing the sheets in different places, but keeping them all horizontal, obtain on each one a figure of lines of force. By comparing these figures one with another, we shall be able to discover how the polar regions are distributed over the magnet.

11. The magnetic field.—The entire space surrounding a magnet, as far as the force exerted has any perceptible magnitude, is referred to as the 'field' of the magnet.

Since for the present at least we are unable to trace the course of the magnetic influence within the substance of the magnet itself, we may consider the field to be confined to the external region. We may also consider it to be limited by a spherical surface enclosing the magnet, and of so great a radius that the magnetic force at all points external to the surface is not perceptible to the tests which we have so far employed. The field occupies the hollow region which lies between this spherical surface and the external surface of the magnet, a remark which is of importance in view of subsequent developments. As our means of detecting magnetic influences become more refined, the outer limits of the field become more extended; and strictly speaking there are no limits at all, the field extending indefinitely into space.

12. Course of the lines of force in the field of a natural magnet.—To obtain a picture of the field of a natural magnet, we must combine together the line-of-force diagrams obtained in § 10. Each of them evidently gives no more than a plane sectional view of the field. We remark, then, how bundles of lines of force start from the boundary of the field (surface of the magnet) in certain polar regions, and after becoming more or less widely spread out as they run their course, are once more closely crowded together as they re-enter the magnet at other places. Many lines do not turn back again, but appear to lose themselves in remote parts of the field.

The field of a magnet is extended in three dimensions, since it occupies a certain region of space. The term 'field' is hardly a correct expression of our meaning, when we ordinarily think of something two-dimensional—that is, a distribution of lines of force over a surface.

B.—Natural magnets with iron armatures

In studying magnetic relations by means of natural magnets, there are difficulties arising from irregularity of shape, and irregular distribution of polarity. We must therefore find some means of directing the magnetic influences, and concentrating them in small determinate regions; to attain which object we may avail ourselves of the property possessed by soft iron, of taking up and transmitting the magnetic state.

13. **Transmission of magnetic influence by soft iron.**—*Experiment* 5.—Let a rod of soft annealed iron be held in a vertical position by means of a wooden clamp, and then let a vessel filled with iron filings be brought near to its lower end. Even if the filings are brought into contact with the rod, very few of them are found to adhere. If, however, a magnetically active corner of a lodestone be held touching the upper end of the rod, a tuft of filings will remain hanging to the lower end, the separate strings of particles diverging from one another. It is thus seen that in the present case magnetic lines of force issue from the lower end of the rod, and these have been transmitted downwards through the iron from the natural magnet above. When the magnet is removed, the iron filings fall from the rod.

14. **Magnetic permeability.**—We shall better understand the transmission of magnetic effects through the soft iron if we apply soft iron bars to the two poles of a natural magnet (figs. 2a, 2b).

Experiment 6.—Place a piece of lodestone on a horizontal plane, uniformly sprinkled with iron dust, arranging it so that two poles may lie as close together as possible, producing the line-of-force diagram (fig. 2a). There are poles

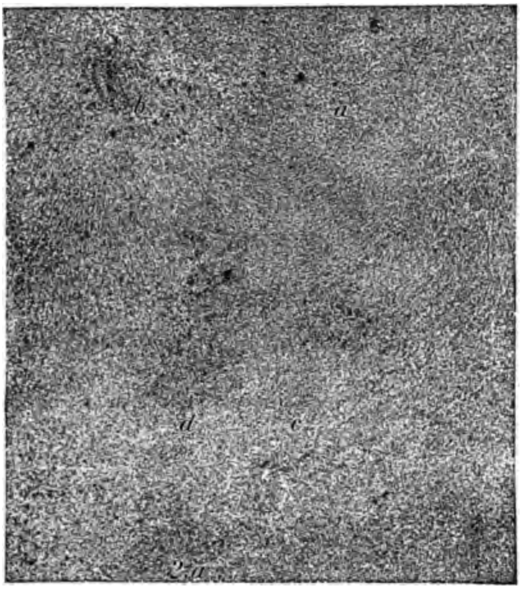

FIG. 2a

above *a* and *b* as is shown in the figure by the numerous lines curved downwards, and passing from one point to the other, though not very easily discernible. The lines of force which diverge from *a* converge again towards *b*. At points more remote from the poles, such as *c* and *d*, the directing influence of the magnetic force is no longer appreciable.

Now place the lodestone on a second piece of paper as thickly dusted over with iron as the first. At the two poles (a^1 and b^1, fig. 2b) lay two elongated pieces of soft iron upon the paper, underneath the magnet, and direct them so that their free ends c^1 and d^1 approach one another more and more closely, actual contact between them being prevented by a small piece of cork attached to the end of one rod. On tapping the paper, the iron dust becomes arranged as in fig. 2b. A comparison of the two diagrams shows that:

α. The lines of force are attracted to the iron rods, many of them beginning or ending on the rods.

β. In particular, the lines of force proceeding directly from the poles *a* and *b* appear to be collected together and transmitted by the iron.

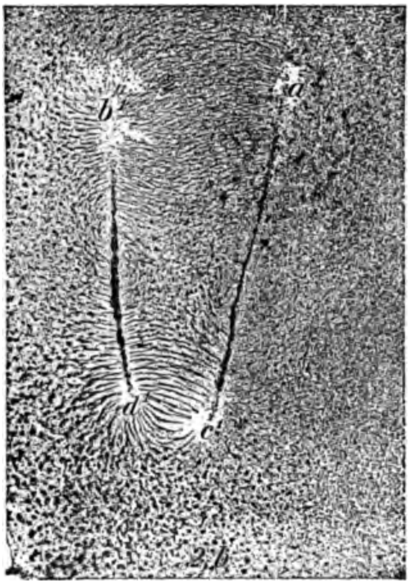

Fig. 2b

γ. Accordingly the number of lines of force passing across from one iron rod to the other, but especially the number of lines issuing from the free ends c^1 and d^1, is much greater than it would be at corresponding parts of the field, when no iron is present.[1]

The magnetic effects which arise from the poles *a* and *b* are rendered perceptible at more distant points of the field c^1, d^1 when they are transmitted there by means of soft iron. Borrowing a term from an analogous problem in fluid motion, we say that the iron has considerable *magnetic 'permeability.'*

15. Magnetic permeability of different substances.—*Experiment* 7.—Let experiment 6 be repeated with rods of wood, glass, copper and brass of the same size as the iron rods. The influence of any of these on the transmission of magnetic influence from the lodestone is not appreciable, the substances in question having a permeability far less than that of iron.

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[1] The little rods of iron used in producing fig. 2b had previously acquired some magnetism of their own.

Under the influence of very strong magnetic fields, however, these substances exhibit in a slight degree properties which are in some respects similar to those of soft iron.

16. Natural magnets fitted with pole-pieces.—The high permeability of soft iron enables us to modify the field of a natural magnet in a convenient way. The polar regions of a lodestone are furnished with closely-fitting iron pieces which are continued in the form of massive iron blocks, and from these very powerful effects are obtained.

The polarity is transmitted to the exterior of these so-called 'pole-pieces,' which serve to concentrate the magnetic effect.

Experiment 8.—Let one of the natural magnets used in experiment 1 be furnished with pole-pieces in the manner described above. On dipping the points of the pole-pieces into iron filings, the adhering tufts will be found to be much longer and stronger than could be obtained at any part of the magnet in its natural condition.

Experiment 9.—Let a magnet furnished with pole-pieces be supported in a wooden stand, with the ends of the pole-pieces facing upwards. To effect this a block of wood, with a cavity cut to receive the magnet, is fixed to a board of sufficient width, while four wooden supports, attached to this board, serve to carry a wooden frame, whose upper edge is at the same height as the pole-pieces of the magnet. The sheet of paper or of glass on which the magnetic pictures are to be formed is laid upon this frame, and when the experiment is not proceeding a piece of soft iron is placed joining the two poles. The line-of-force diagram in fig. 3 shows how the lines of force pass from the one pole (n) to the other (s). The crowding together of the lines near the poles shows how the magnetic force is concentrated at these places.

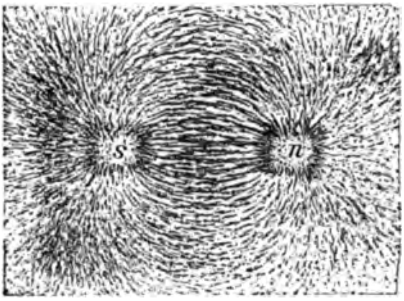

FIG. 3

We shall find it convenient in future to mark the poles in some definite manner, one with red and the other with blue (preferably with enamel colours). Accordingly the lines of force will be spoken of as issuing from the red pole (*n*) and entering the blue one (*s*).

17. **The keeper.**—If a piece of soft iron, whose length is equal to the distance between the pole-pieces, is brought near to the latter, many of the lines of force leave the surrounding space, and pass through the iron. Just as chains of little iron particles were previously held fast, so now the entire mass of iron will be supported, when we arrange it as a bridge connecting the pole-pieces. On turning the magnet round, the soft iron is found to remain hanging to it, the attraction due to magnetic influences overcoming gravity. The piece of soft iron is called the *keeper*, because the closing of the magnetic circuit in the manner described has an important influence in preserving the properties of the magnet.

Experiment 10.—*Effect of closing the magnetic circuit.*—Let a magnet provided with pole-pieces be laid in a horizontal position, then form and fix the line-of-force figure just over the pole-pieces. Next place the keeper near to the pole-pieces, form another figure, bring the keeper nearer, once more form a figure, and so on. A comparison of the different figures shows how the keeper gathers together and transmits through its substance the lines of force which pass from pole to pole. The nearer it is brought to the magnet, the more numerous are the lines of force which enter and leave it, nearly all the lines passing to the end of the keeper from the nearest magnet-pole across the shallow intervening gap, where they follow approximately the shortest possible path. If the keeper is brought into actual contact with the poles, it is held fast by them.

C.—Artificial magnets of various forms and their fields

We should make but slow progress in the knowledge of magnetic phenomena if all our investigations were to be made with natural magnets, with or without pole-pieces. In the first place because we should be limited in our choice of the most suitable shapes, and in the second place because natural magnets are always weak, and diminish in strength as time goes on. But as we know how to produce magnets of any desired shape and to strengthen their magnetic effects, we have thus the means of greatly extending our researches.

When we have described the most important forms of artificial magnets, it will be necessary to discover the distribution of their fields, which we shall investigate as before by means of figures of iron filings. In the applications which we have to make of these magnets, the disposition of the lines of force is of the utmost importance.

18. **Communication of magnetic properties to steel.**—Iron filings, placed in the field of a natural magnet, were found in previous experiments to acquire magnetic properties which enabled them to attach themselves to other iron particles; but they lose this property as soon as they are removed from the field. It is, therefore, of the greatest importance to obtain iron in such a form that, when magnetic properties are given to it, it shall be able to retain them permanently. This is accomplished by the succession of operations by which ordinary iron is transformed into steel.

Experiment 11.—Out of a considerable number of stout steel knitting-needles, select one which, on being dipped in iron filings, retains none of them adhering to it, and is therefore non-magnetic.

If no such needle can be found, then one must be demagnetised. This is to be done by heating a feebly magnetic needle to redness, and then suddenly chilling it by immersion in cold water. The needle will then have lost its magnetism almost completely, while the steel of which it is formed will be as hard as before.

Now fix in a vertical position a soft iron wire, made specially soft by annealing, and having the same thickness and length as the needle. On bringing the pole-piece of a lodestone into contact with the upper end, the phenomenon already described will occur; the lower end of the wire will be able to support iron filings. On removing the pole-piece, the filings fall off again.

If the same experiment is repeated with the steel knitting-needle the lower end is able to support iron filings, but to a much less extent than was found in the case of the iron wire. But when the pole-piece is removed from the upper end of the needle, the filings do not drop off; the needle retains the property communicated to it by contact, it is permanently converted into a magnet. This communication of the magnetic property to steel is called magnetising it, the result being a so-called 'artificial magnet,' or 'permanent magnet.'

This experiment shows a two-fold difference between the behaviours of iron and of steel. Iron assumes the magnetic property more readily than steel, but is also more easily demagnetised; steel, on the other hand, though harder to magnetise, retains the magnetic property which it has once acquired. The difference exhibited by iron and steel, in respect to the ease with which they change their magnetic condition, has been referred to an influence which tends to the retention of magnetism, and which has been called 'coercive force.' Iron has thus a small, and steel a considerable, coercive force [hysteresis].

19. **Production of artificial magnets.**—Since in the communication of the magnetic property it is of fundamental importance to make the lines of force pass as completely as possible through the steel bar to be magnetised, more powerful magnets will be obtained when the contact is made to extend over the entire bar; that is when the bar is stroked along its entire length with one poie or with both.

(*a*) *Single touch.—Experiment* 12.—Lay on the table an unmagnetised bar of steel (for example a knitting-needle), and, holding one end firmly, stroke the needle several times along its entire length in the same direction with one pole-piece of a natural magnet, for example that which is painted red. When iron filings are strewn near the needle, they arrange themselves in chains. Lines of force leave and re-enter the needle, especially near the ends. If the ends are dipped in iron filings, tufts of these latter will be left hanging to them.

If a steel bar has acquired magnetic properties from a natural magnet, it can further be used to magnetise another steel bar.

Experiment 13.—Stroke a knitting-needle from end to end several times in the same direction with a magnetised knitting-needle. The needle so treated will also become a magnet, and in this case, too, the magnetic activity will be manifested chiefly at the ends.

(*b*) *Magnetisation by double touch.*—On examining the line-of-force diagrams produced from bars magnetised in the manner described, it will be seen that here, as in the case of natural magnets, the lines of force issuing from one end re-enter the magnet at the other. This implies the contrariety of properties to which we have given the name 'polarity.' It will therefore be better, instead of stroking the entire length of the bar with the same pole of the magnetising magnet, to stroke one half with the pole which is painted red, and the other half with the pole which is painted blue.

Experiment 14.—A knitting-needle of the same sort as that used in experiment 12, after being tested in the manner already described, to make sure that it is not already magnetised, is held by pressing upon it at the middle with a finger of one hand, while a natural magnet provided with pole-pieces is drawn over it with the other hand, one half of the needle being stroked with one pole and the other half with the other pole, alternately, until each half has been stroked about thirty or forty times.

(*c*) *Magnetisation by divided touch.*—Since the lines of force are especially numerous between the pole-pieces of a magnet, we may also bring the two pole-pieces simultaneously into contact with the steel bar to be magnetised, and move them backwards and forwards, from end to end of the bar. This method is especially applicable when several bars are to be magnetised at the same time.

Experiment 15.—Place four or more unmagnetised knitting-needles end to end so as to form a closed figure, and let them be fixed in this position. Then let the two pole-pieces of a natural magnet be drawn repeatedly round the circuit of needles, always in the same sense. Magnetic effects will afterwards be discovered at both ends of every needle.

As regards the methods which may be used for magnetisation, the description already given must suffice; we will only remark here that all the steel bodies described in the sequel are supposed to have been strongly magnetised by one or other of these methods. We shall afterwards see that there are much more effective methods of magnetisation, depending on the employment of magnetic fields due to currents.

20. **Immaterial nature of magnetism.**—When a body is magnetised, no change is produced in its weight.

Experiment 16.—Weigh a bar of good steel upon a sensitive balance which has no iron or steel parts. Then magnetise the bar as strongly as possible and weigh it again. The weight has remained unaltered.

This experiment must only be taken as a type of a large number which have been performed in various ways, and which have all given negative results.

It follows that when a body acquires the magnetic property, nothing of a material nature is added to it. Thus magnetisation can only consist in the rearrangement of the matter already existing in the body, or in giving to this matter some kind of motion, which constitutes the distinction between magnetised and unmagnetised bodies. The motion in question must further be of such a kind as not to involve motion of the bar as a whole in any one direction.

We shall further see how these conditions are satisfied by a cyclical or rotatory motion in the smallest parts of a magnet and of the field surrounding it, and what are essentially the transformations of energy involved in the process of magnetisation.

21. **Bar magnets.**—When we magnetise a steel bar of sufficient length, and having a cross-section of any form, we obtain a bar magnet. The length should be very considerably greater than the greatest measurement in any perpendicular direction. Magnetic effects are exhibited almost exclusively at the ends. Fig. 4 shows a line-of-force diagram in a horizontal plane parallel to the axis of such a magnet; from it we see that the extremities are polar

regions, especially the most outlying parts of all, N and S, though no particular point can be specified as the exclusive origin of the lines of force.

The lines of force issuing from one polar region into the surrounding field curve round in wider and wider circuits until they reach the other polar region, where they terminate.

We find, therefore, the same properties which led us to distinguish between the two polar regions of a magnet with pole-pieces, when we painted them red and blue.

At the middle of the bar there is a region J where hardly any lines of force begin or terminate; the magnetic force in the immediate neighbourhood of this place is determined only by the lines which pass from the red to the blue end, and which here run parallel to the bar. This portion of the bar, then, contributes nothing directly to the strength of the field; it is called the *indifferent zone.*

If we give the bar magnet a half turn, more or less, about its axis, we still obtain the same line-of-force diagram in a plane parallel to the axis. Hence it follows that the field is symmetrical with respect to this axis (approximately when the cross-section is angular, accurately when it is circular). If a round bar magnet is supported in a vertical position, the line-of-force diagram fig. 5 will be obtained in a horizontal plane. This figure corresponds, therefore, to a plane cutting through the field perpendicularly to the axis of the magnet, all the lines being seen to run radially outward. To form a picture of the true disposition of the field (in three dimensions) we must imagine a series of planes, each drawn through one of these lines and the axis of symmetry of the magnet, and each corresponding to a distribution of lines like that depicted in fig. 4.

Since many lines of force originate at the circumference of the end of the magnet, there is in the diagram a correspondingly denser aggregation of iron particles, forming a black ring in the middle. The neighbouring portions of the diagram, within and just without this ring, have very few particles remaining, the

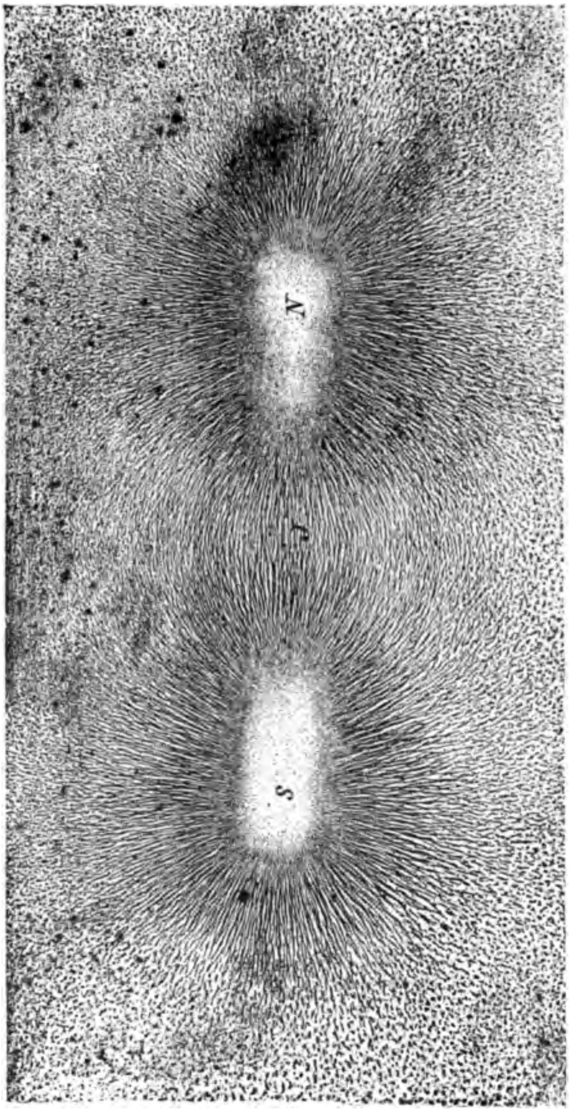

FIG. 4

greater part having moved away to the annular region in question. Similar appearances are always to be observed in line-of-force figures when the magnetic force has a very strong component in the plane of the paper.

In the case of longer and thinner bar magnets, it is found that the lines of force diverge from two points whose distance apart is equal to about five-sixths of the length of the bar.

The case of very long thin bars is especially important. If such a bar be magnetised, iron filings will be found to attach themselves to the ends almost exclusively, assuming the form of nearly spherical tufts. Line-of-force diagrams show that directing forces are only appreciably exerted on iron particles lying near to the extremities, or poles. The lines of force in the neighbourhood of each pole follow a course which is not appreciably influenced by the more distant pole, the indifferent zone is much longer, and the particles surrounding either pole are not perceptibly acted upon by the other pole. In this way we are able to arrive at the conception of a single isolated pole; a conception which we shall frequently have occasion to employ.

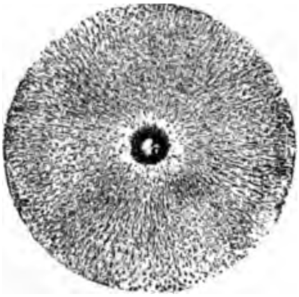

FIG. 5

The field at either end of a long bar magnet of this kind is called 'unipolar.' For almost all forms of the cross-section of the magnet the lines of force proceed radially, as in fig. 5.

22. **Compound magnets.**—As we have already explained, the magnetic effect of a magnet depends upon the number of lines of force which issue from it: the greater the number of lines proceeding from a magnetically active place, the greater their effect. To increase the effect due to a bar magnet, we may combine it with similar bars, magnetised in the same way; since the lines of force all diverge from one end, and converge again to the other end, the bars must

be so arranged with regard to one another that all the ends having the same kind of magnetism are together; that is, all the poles marked red are to be at one end, and those marked blue at the other end. (In § 27 we shall explain a simple criterion for distinguishing between the two kinds of polarity.) Combining in this way a number of bar magnets, each of which has been previously magnetised as highly as possible, we obtain a compound magnet.

The magnetic effect due to a compound magnet is not proportional to the number of rods of which it is built up. There is no simple relation between the weight of the compound magnet and its strength; the explanation of this being found in the influence exerted upon one another by magnets laid side by side with like ends together.

23. **Magnets of special forms.**—From bar magnets we may pass on to magnets of some special forms, which it will be convenient to mention in this connection, some being of great practical importance, while some are more of theoretical interest, and are destined to play a most important part in the sequel. Very thin magnets may be made out of clock-spring. If we cut from a long thin strip of steel a piece in the form of a very elongated rhombus, and magnetise it by stroking with another magnet, so that the poles are at the acute corners, the result is a so-called *magnetic needle*.

Again, let a short thick bar of steel be magnetised perpendicularly to its axis. The transversely magnetised body so obtained has polar regions on the two sides of its bounding surface. If we imagine the length of the bar to be made excessively small, we have a plate with poles at opposite points of its circumference. Small bell-shaped pieces of steel tube, closed at one end, are sometimes magnetised so that their poles are at opposite sides of the cylindrical bounding surface. A steel ring may be magnetised so as to have poles at opposite ends of a diameter. All these forms are of importance in connection with certain electrical and magnetic measuring instruments, which will be described later.

If a rod is strongly magnetic at both ends, and if we sufficiently reduce the length in the direction of the axis, we finally obtain a plate which has a distribution of magnetism on each face, so that the lines of force proceeding from one face bend round, embracing the rim of the plate, and finally reach the other face, where they terminate. A magnet of this kind is called a 'magnetic shell.' We may, if we choose, regard a long bar magnet as made up of a succession of magnetic shells placed face to face; all the faces of one kind looking in the same direction.

We shall find in Section II. that magnetic shells are of great theoretical importance, on account of the analogy between them and certain electro-magnetic arrangements (circuits in which currents are flowing). In view of the important applications which we shall afterwards have to make of them, we shall now construct circular, square, and rectangular cardboard models of magnetic shells, one side being covered with red paper and the other with blue.

24. **The horse-shoe magnet.**—Since the effects produced by a bar magnet are strongest at the ends, where the lines of force are most numerous, we may conveniently obtain stronger fields by bending a bar into the form of a horse-shoe or an incomplete ring, so as to bring the two ends near together. The ends of the horse-shoe are then finished by filing, so as to make their terminal faces lie in one plane, and the bar is magnetised along its entire length. If a horse-shoe magnet of this kind be supported in a vertical position, with its ends upwards, and if a line-of-force figure be then formed upon a sheet of paper held just above, the diagram obtained will be similar to that shown in fig. 3. In a plane drawn through the curved axis, that is, in the 'median plane' of the magnet, or in a neighbouring parallel plane (fig. 6), the lines of force proceeding from the ends *n* and *s* are seen to be closely crowded together, while in the space between the limbs of the horse-shoe they pass from one side to the other by very nearly the straightest and shortest path. Outside this space the lines are bowed out into wider and wider arcs, those proceeding from the outer

parts of the extremities being distributed over the more remote parts of the field. The nearer we approach to the bend of the horse-shoe, the fewer are the lines of force emerging from the body of the magnet, and finally the lines

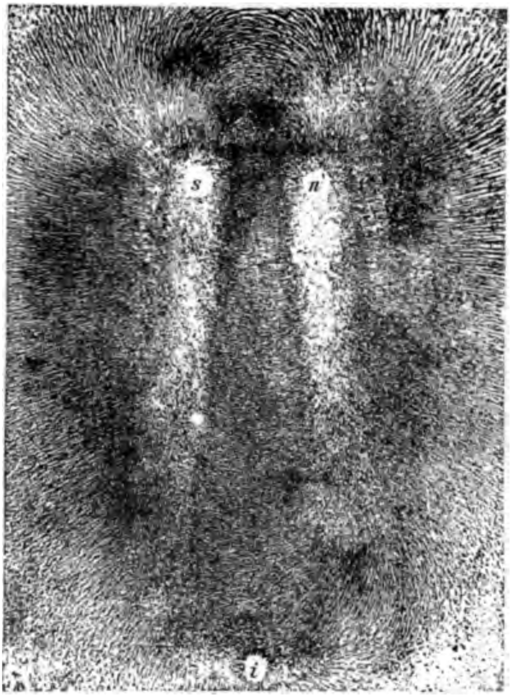

Fig. 6

run parallel to the bend, the particles of iron lying just outside the bend being subject to scarcely any directing force. Here, then, just at the bend, is the indifferent zone of a magnet of this form.

It will be seen that in fig. 6 there is a point towards the lowest part of the right limb where several lines of force run together; in this position there is, as it were, a pole of subordinate strength, corresponding to another just below *i*, which is harder to distinguish. Such irregularities (' consequent poles ') are often present when the magnetisation is not quite uniform.

As in the case of bar magnets, we can obtain a horse-shoe magnet of greater strength by binding together a number of pieces of thin steel plate, cut in the form of a horse-shoe and separately magnetised. If a piece of soft iron (the keeper) be brought into contact with the two ends, it will be strongly held by the magnet (§ 17).

Here again the lifting strength is not proportional to the number of separate magnets used in building up the combination. The formulæ which have been obtained involve quantities depending on the quality of steel and the method of tempering and magnetising it. They need not be further considered in this place.

25. **Determination of the spacial distribution of lines of force in strong fields.**—When we have to deal with strong magnetic forces, for example those due to compound magnets, we can determine directly the course of the lines of force, not only in a plane, but in three dimensions.

(*a*) *Fixation of the finest iron dust in the form of the lines of force by burning.*—Sprinkle the finest iron dust between the poles of a powerful magnet; the particles adhere together and build up a stable bridge between the two poles, the separate chains following the form of the lines of force (very approximately at least where the force is strong, though in weaker parts of the field the disturbance of form arising from gravity may be considerable). On setting fire to the iron it becomes oxidised, and coheres so as to form a solid mass which reproduces the structure of the field of force. We may then remove the slag, and, by making sections of it, follow out the structure of the field.

(*b*) *Fixation by imbedding in gelatine.*—Let coarse iron particles be imbedded in a semi-fluid body, such as gelatine, which does not interfere with the rotation of the particles or with their displacement through small distances, and leaving the whole exposed to the influence of a magnet, allow the gelatine to harden.

Experiment 17.—Suspend a horse-shoe magnet with its poles pointing downwards, and arrange a card-board box so that the poles dip into it. Dissolve some fine transparent gelatine, then warm the solution and stir in a considerable quantity of iron filings. Any fine dust which may be present, and which would render the gelatine opaque, should be removed by sifting it away. The gelatine solution is now to be poured into the cardboard box, so as to surround entirely the lower part of the magnet, and there allowed to stiffen.

On removing the cardboard box, and cutting away the outer layers of gelatine in which the filings are rusted, we obtain a transparent mass in which the course of the lines of force can be clearly followed from every side, especially when a strong light is allowed to shine through. The iron filings in the interior of the mass do not rust very quickly.

If the form of the lines of force is to be projected by means of the lantern, it is better to immerse the particles of iron in some viscous transparent fluid.

Let *ferrum redactum* be stirred into glycerine or paraffin oil, contained in a thin glass vessel, which is then placed horizontally in the beam of a lecture lantern, so that an image of the iron particles is projected on the screen. When a strong magnet is brought near, the particles arrange themselves along the lines of force; and if a piece of soft iron be immersed in the trough, the lines of force are seen to become concentrated in its neighbourhood.

26. **Uniform magnetic field.**—Generally speaking, the lines of force are curved; but if we examine the field of a horse-shoe magnet, we find that there are places between the limbs of the horse-shoe where the course of the lines is very nearly rectilinear. Here they are evenly distributed with the same degree of closeness throughout the field. Fields of this kind are of great importance and are called 'uniform fields.'

Such fields are similar to the earth's gravitational field within a restricted region, for example in a room; there the weight of a body is everywhere the same, and at all points the direction of the force is vertical, the gravitational lines of force being parallel to one another.

D.—Magnets with one degree of freedom

All the magnets so far employed we have supposed to be held fixed. We shall now consider the case where the axis joining the poles of the magnet is free to rotate in a horizontal plane about its middle point.

In the case of a horse-shoe magnet, this is accomplished by hanging it up by a thread attached to the middle of the bend. Bar magnets may be suspended by means of a brass loop furnished above with a hook to which a thread is attached, the adjustment being so made that the magnet swings freely in a horizontal plane. The suspending threads must first be untwisted by leaving a weight suspended from them. Small light magnets (magnetic needles) are sometimes fitted at the middle with a cap in which a hollow piece of agate is set, so that the needle may be supported with as little friction as possible on a vertically directed steel point.

We thus give to the magnet 'one degree of freedom,' as it is called; that is to say: Out of the infinitely numerous positions which the axis could occupy in space, we confine ourselves to those (singly infinite in number) which the axis could occupy in a horizontal plane when one point of the magnet is fixed. Further on we shall consider the effect of giving more than one degree of liberty to the magnet, which we shall suppose free to set with its axis in any direction, horizontal or otherwise; the point of suspension alone being fixed relatively to the earth's surface.

27. **South-to-north direction of a freely turning magnet.**— A magnet, suspended by a thread or supported upon a steel point, can turn freely in any direction in a horizontal plane; but there is an experimental law, which is of fundamental importance in regard to magnetic phenomena, and which may be stated as follows:

If the axis of a magnet is free to turn in a horizontal plane, it will set itself in a determinate direction.

If the magnet is displaced from this position, and left free to move, it executes vibrations of gradually diminishing amplitude, and finally comes to rest in the same position again. If we examine the phenomenon in relation to the points of the compass, we find that one pole points to the north (this is the 'north pole' or north-seeking pole), the other pole to the south (south pole). This furnishes a means of specifying the poles of a magnet without ambiguity, and of distinguishing one from the other.

For example, we may verify in this way that in the method of magnetisation by double touch (§ 19 b) the end rubbed with the north pole of the exciting magnet becomes a south pole, and *vice versa.*

The phenomenon of the south-to-north pointing of a suspended magnet was made known to the world in the seventeenth century, though as early as the year 121 A.D. it was mentioned in a Chinese dictionary (PESCHEL).

We shall consider later the significance of the phenomenon as touching the physical constitution of our planet.

We shall distinguish all north poles by red, and south poles by blue (*cf.* § 16). In order to render the polarity immediately recognisable in the figures, north poles are ordinarily shaded perpendicularly to the magnetic axis, and south poles parallel to this axis (AIRY).

Like poles are those which tend to point in the same direction at any given place; unlike poles are those which tend to point in opposite directions.

CHAPTER II

THE MUTUAL ACTION OF TWO MAGNETS

A.—Mechanical force exerted by a fixed upon a movable magnet. Direction of the lines of force

In this chapter we shall become acquainted with new properties of the lines of force, relating to the effects produced upon a movable magnet under their influence. For the present we leave out of account the system of lines of force arising from the movable pole, using this only to investigate the course of the lines of force in the field of a fixed magnet.

28. **One pole of a long straight movable bar magnet in the field of a fixed magnet; mechanical effects.**—We saw above (§ 21) that in long thin bar magnets (regularly magnetised) the magnetic effectiveness is concentrated almost exclusively at the ends. This fact enables us to subject a single pole to the influence of a fixed magnet, the movable bar magnet being supposed made so long that the second pole lies outside the region where the field of the fixed magnet is appreciable.

Experiment 18.—To one end of a long strongly-magnetised steel wire, a loop or eye of brass is affixed, so that the steel wire may be hung up by a thread, and will then rest in a vertical position with its lower end just above the table.

The steel wire should not be too flexible, and must be made quite straight. The suspending thread must be untwisted, thin and not too long, or the wire will too easily acquire an oscillatory motion. It will be best, indeed, to load the wire somewhat at its lower extremity, which should also carry a white paper cross so as to be easily visible from a distance.

Upon the table magnets are placed, the corresponding line-of-force figures being formed upon a sheet of paper placed beneath them. It will be found that the magnets exert upon the lower end of the wire (which is freely movable in all horizontal directions) a force whose direction is determinate. This force produces a bodily displacement of the suspended magnet, which is caused to deviate more or less widely from its vertical position of rest.

We proceed to investigate the nature of the mechanical force exerted on a magnet pole in some special cases.

(*a*) The north pole *n* of the movable magnet being downwards, we bring under it:

(*α*) The north pole N of a long powerful bar magnet; the pole *n* will be repelled from all sides, and if we force it back to its original position, it will move away again as soon as we let go of it. Some amount of work, then, must be expended in forcing the pole *n* towards the pole N.

If we examine the direction in which the pole *n* is displaced, we shall find that it is always coincident with that of the lines of force passing through N.

The north pole N of a fixed magnet always repels the north pole n *of a movable magnet along the lines of force proceeding from N.*

Now turn the fixed magnet round, end for end, so that

(*β*) The south pole S is just beneath the movable pole *n*. The phenomena are completely reversed. There is a mechanical force urging the suspended wire towards the fixed magnet, and work must be expended in order to move the pole *n* out of the field of the pole S. The force to be overcome follows everywhere the direction of the lines of force.

The south pole S of a fixed magnet always attracts the north pole n *of a movable magnet along the lines of force proceeding towards S.*

(*b*) Next reverse the direction of the suspended magnet, the attachment of the suspending fibre being transferred to *n*, so that the south pole *s* is hanging downwards, and we shall have reversed the mechanical forces corresponding to the positions (α) and (β) of the fixed magnet.

The pole *s* will be attracted by the north pole N of another magnet, and repelled by the south pole S. The direction of the displacement of *s* will in every case be along the line of force due to the fixed magnet and passing through the position of *s*.

The phenomenon is thus a polar one in respect to each of the magnets concerned in the mutual attraction or repulsion.

Experiment 19.—Place a short bar magnet with its middle point beneath the movable magnet, so that its indifferent zone is intersected by the vertical through the point of suspension of the steel wire. The wire leaves its vertical position, and moves towards one end of the bar magnet. If it is forced towards the other end, and then left free to move, it returns past the indifferent zone, back to the position it had when undisturbed, its path following the general direction of the lines of force (*cf.* fig. 4). The pole *n* is always urged in the direction from the pole N to the pole S of the fixed magnet; the pole *s*, on the contrary, from S to N.

Experiment 20.—If we bring a horse-shoe magnet into various positions beneath the movable pole, this latter is found to move in a manner similar to that observed in the last experiment. Between the limbs of the magnet, where the lines of force pass nearly straight from side to side, as well as in the space outside the magnet, where they bend round in wider and wider arcs (fig. 6), the displacement of the suspended wire follows the direction of the lines of force Again, for any given disposition of the fixed magnet, the poles *n* and *s* will be urged in opposite directions.

Instead of freely suspending the steel wire by a thread, we may allow it to float (Weiler).

Experiment 21.—A long thin bar magnet, such as a magnetised knitting needle, is stuck through a cork, which is floated in a large vessel of water; the needle dipping vertically downward into the water, and having one pole or the other at the free surface. The upper pole is subjected to the influence of bar or horse-shoe magnets, placed upon the rim of the vessel. The needle moves in one direction or the other through the water, according to the end which is uppermost and the polarity of the fixed magnet; the force exerted upon the pole being always in the direction of the lines of force.

All the foregoing experiments support the following important conclusion: *The lines of force have at every point a determinate direction*, which is indicated by the different forces exerted upon north and south poles placed in the field.

Contrariety of direction, such as we meet with here, is expressed by calling one direction positive (+) and the opposite direction negative (−). As we inhabit the northern hemisphere of our planet, we shall reckon positively the force exerted upon a movable north pole placed in the magnetic field; adopting the convention that *the direction in which a line of force proceeds is that in which a movable north pole would be urged along it.*

If we examine the results obtained from the foregoing researches we see further that for all magnets the lines of force issue from the north pole, and pass through the field to the south pole, where they re-enter the magnet.

We had already observed a contrariety of this kind in the properties of the two poles of a magnet, and employed the same notation as that just adopted to distinguish between the issuing and re-entering of lines of force. The experiments since carried out justify the step. The specification of the positive direction along the lines of force involves an arbitrary convention, but once this is agreed upon, the expression of all other relations is determinate.

29. **A short freely movable magnetic needle in the field of a larger magnet.**—Since the north pole *n* of a movable magnet tends to move in the direction of the lines of force, and the south pole in the opposite direction, a short mag-

netic needle with the two poles close together will, if free to turn, set itself directly along a line of force. The lines of force are in general curved, and the magnetic needle points along the direction of the tangent at the place where it is situated.

The direction of the lines of force is thus completely determinable at every point of the field as the direction in which the axis of a short magnetic needle will point, the positive direction of the axis being from *s* to *n*.

Experiment 22.—A good steel sewing needle is suspended from a thin untwisted thread (preferably of fine silk) so as to hang quite horizontal under the influence of gravity. To keep the thread from slipping on the needle, it is fastened with sealing-wax. The needle is then to be magnetised by stroking it with other magnets, care being taken not to alter the point of suspension. We have thus a magnetic needle with two degrees of freedom. It points north and south, and the polarity of the two ends (point and eye) is to be noticed.

If this needle is brought to various positions near to a piece of lodestone, it always assumes a direction which coincides with that of the tangent to the line of force passing through the point of suspension. At some places the point of the needle will be found to point towards the lodestone, at other places away from it. Thus we may discover where the polarity of the lodestone is north-seeking, and where south-seeking.

Such 'exploring needles' were used by Faraday, and are very useful for determining the constitution of complex magnetic fields, being often more convenient than the iron-filing pictures.

We shall always indicate the direction of the lines of force by means of the forefinger.

In the lines of force we meet for the first time with a definitely specified direction, and ordinarily when we wish to indicate a direction, we use the forefinger of one hand or the other. In the field of a magnet, then, we should have to hold the forefinger pointing away from the north pole towards the south pole. In all the models which we shall have to describe, the directions of the lines of force will be indicated by white arrows (cut out of tin for example, or out of white paper).

30. Exploration of the course of the lines of force.—Short magnets with one degree of freedom, such as needles freely turning on points, will serve to determine the course of the lines of force in a plane.

Experiment 23.—Exploration of the lines of force of a large bar magnet.—Let a strongly magnetised steel bar or a compound bar magnet be laid on a roll of drawing paper spread out horizontally, and then bring into various parts of the field a small magnetic needle, mounted on a low stand, so that its height above the paper is about equal to half the thickness of the bar magnet. The needle will everywhere set itself in a determinate direction, which may be recorded by marking upon the paper the points vertically beneath the two ends of the needle, joining them by a line, and affixing an arrow-head to show in which sense (*s*, *n*) the line is to be taken. If this operation is performed at many points the system of arrows will give a very clear idea of the course of the lines of force in the field. The results may afterwards be verified by means of iron filings.

This method of recording isolated positions of the needle is in some measure incomplete, since it does not give directly the actual course of any single line. But if to the upper end of a lead pencil we attach a metal point, upon which a magnetic needle can turn freely in a horizontal plane, the course of a line of force may be followed through the field by so guiding the pencil that the projection of the needle on the paper is always tangential to the last drawn element of the curve.

A very convenient and simple arrangement for this purpose is shown in fig. 7. A round brass foot A, whose under surface is plane, has a hole in the middle into which is inserted a brass tube B, five centimetres long, and provided at its upper end with a needle-point C. Inside the tube is a piece of lead pencil D, which is pressed downwards by the spiral spring E, but is kept from falling out by a constriction of the lower end of the tube, only the point F being able to protrude. When the pencil is to be renewed, the top C of the tube can be removed. On the pointed end of C, the small bar-magnet G, with poles marked N and S, can turn freely in a horizontal plane. When not in use, the instrument stands on a cork base, which has a small depression to receive the point F.

If we desire to map out the lines of force of a magnet (for example, of a horse-shoe magnet), we place it over a sheet of paper, supporting it on a wooden block so as to raise its 'median plane' to the same height as the magnetic needle G. By looking vertically down upon G, it is easy so to move the foot A, that the tangent to its path is always parallel to G. We have thus a very fairly accurate graphic construction for the course of a line of force, so far as it corresponds to the median plane of the magnet under examination.

One of the most comprehensive collections of such systems of curves, for magnets of the most various forms (bar-, horse-shoe- and ring-magnets) was brought out in 1844, by J. E. HERGER, with a preface by ERMAN. It was published by E. PÖNICKE and Son, of Leipzig, and contains thirty-seven large diagrams of lines of force, in thirty-one folio plates.

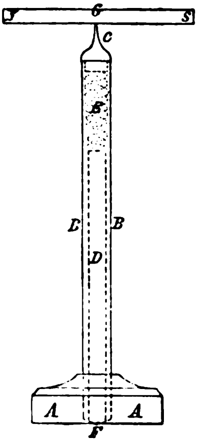

FIG. 7

Specification of the magnetic force as a directed quantity.—Two data are necessary for the complete specification of the effect produced upon a movable pole when placed in a magnetic field.

(1) A determinate *direction*.—This will be at each point coincident with that of the lines of force passing through the point, and is to be determined by drawing the corresponding tangent to the line of force, or more simply by taking an element of the curve which includes the point itself and is short enough to be regarded as straight. The complete specification of a direction includes also a determinate *sense*. Lines of force at a given position in space may lie along the same curve and yet differ from one another in direction, a north-seeking pole being urged one way along the one line of force, and the contrary way along the other. It is therefore essential to know the sense in which a line of force proceeds, as well as the form of the curve which it follows.

(2) A determinate *strength.*—We have already seen that the magnetic effects in the more remote parts of the field are weaker than those in the neighbourhood of the poles. We shall learn later from quantitative experiments that the same is true of the mechanical force exerted upon a movable pole. Thus the effects observed at different places in the same field differ also in intensity.

Quantities which have direction (including sense) and magnitude are called '*vectors*'; they may be represented geometrically by a straight line of definite length drawn from a point in a definite direction. *The magnetic force is a vector, or directed quantity.*

The simplest vector quantity is the displacement of a point from one position to another along a straight line of determinate length drawn in a determinate direction; hence the name (*vehere*=to carry). Velocity and acceleration are also vectors. The part played by vector quantities in physics is very important. We shall find that rotations about axes are also included in the same category.

32. Faraday's representation of the distribution of magnetic force by means of the lines of force.—The lines of force furnish us with sufficient data for the complete determination of the magnetic force at each point of the field. The course followed by the lines shows immediately the direction of the force; while by introducing a movable north pole into the field, we can discover which is the positive sense along any line of force. It would seem, perhaps, more difficult to express the *magnitude* of the magnetic force by means of the distribution of lines. But the density or closeness with which these lines are packed together in any given part of the field may be made to furnish a measure of the intensity of the corresponding force. This density is most simply measured by the number of lines of force intersecting some area whose plane is at right angles to their course, for example, the number of lines per square centimetre; this last-named quantity being called the *field intensity* at the corresponding point.

It will be shown later on that this mode of representation leads to an absolute and not merely to a relative measure of the magnetic force. As the principle of the method is somewhat unfamiliar, we shall here introduce a simple example which may help to make it clearer.

Experiment 24.—*Examination of the distribution of the lines of force in three dimensions.*—Cut down upon one of the gelatine blocks described in § 25, and count the lines of force which intersect the different sections.

In the neighbourhood of the pole of the magnet, a square centimetre of surface, taken perpendicularly to the direction of the lines of force, will be marked with many points where these lines intersect it (considerable strength of field); while in places more remote from the pole the intersections per square centimetre will be few (small strength of field). The numbers thus obtained are not directly comparable, because the formation of the chains of iron filings is influenced by numerous accidental circumstances; the method will only become exact when we suppose the lines of force to be built up and distributed according to a definite law. The example is merely intended to illustrate the possibility of using the method to show graphically the magnitude of the magnetic force.

B.—Mechanical action between two movable magnets.

We shall next consider the case where two magnets, subject to one another's influence, are both movable. One of them (M_1) we shall suppose to have one degree of freedom, the constraints imposed upon it being such that it can only move to and fro in the direction of its axis, or at right angles to it; or we shall suppose it suspended by a thread or mounted on a needle-point, so that it can turn freely in a horizontal plane. In its position of equilibrium in the last-mentioned case, it will point north and south. The second magnet is to be entirely free; it is to be held in the hand, so that it can be approached to the first magnet from every possible side.

We shall now dispense with the condition that the distance from pole to pole of a magnet is to be very great; on the contrary we shall suppose both pairs of poles to take part in the effects observed, and shall seek to determine how the lines of force of the one magnet are influenced by those of the other.

33. Reciprocal action of the fields of two neighbouring coaxal bar magnets.—To render a bar magnet freely movable in the direction of its own axis, it is placed in a little boat made of sheet-brass (fig. 8) and suspended by threads from the ceiling.

A rectangular piece of sheet brass about 12 cm. long is bent round the bar magnet m_1, which is about 25 cm. in length and of circular section. At the four corners S-shaped hooks are inserted into holes drilled in the brass. The hooks are fastened to long thin threads, which are passed in pairs over other hooks fixed in the ceiling. Beneath one end of the bar magnet is a block of wood K, which carries a pointer Z.

Experiment 25.—Let the pole n of the magnet m_2 be brought axially towards the pole n of m_1; m_1 recedes from m_2, as indicated in the figure by the arrows. If the pole s of m_2 is approached to the pole s of m_1, a similar repulsion is observed. The receding of the magnet m_1 causes it to rise somewhat above its original height, so that a certain amount of work is necessarily done against gravity.

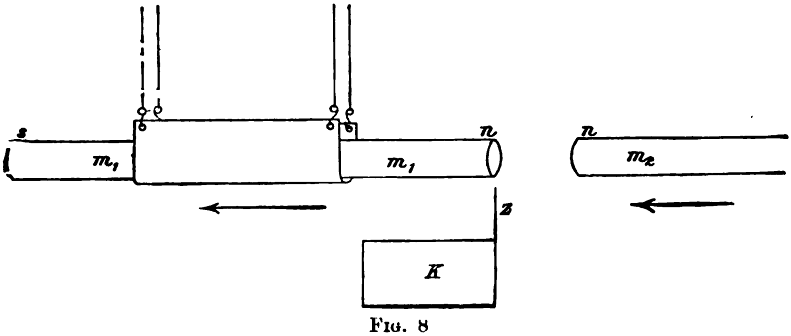

Fig. 8

The field due to a magnet-pole is in some degree *impenetrable* by the like pole of another magnet.

When we stretch out our hand, and suddenly feel a resistance to its movement, we say that there is 'something there.'

To this something, which resists the penetration of other bodies into the space which it occupies, we give the name of 'matter.' The effect observed is also summed up by saying that the region in question is filled with a certain form of energy, which resists a diminution of volume. The knowledge that a magnetic field is a region filled with energy of a special kind is of very great importance.

The phenomenon observed in our experiment is most simply described by saying that *the fields due to poles of the same name repel one another.*

Experiment 26.—Let the pole s of m_2 be brought near to the pole n of m_1 and *vice versa*: the magnets will be attracted together. *The fields due to poles of unlike name attract one another* provided they are brought near enough to exert mutual influence. The masses of the bars themselves have no part in the mutual action; the origin of the effect must lie entirely in the lines of force emerging from or converging to the poles; for in all the foregoing experiments these have shown themselves to be the true cause of magnetic effects. The masses of 'magnetic matter' thus appear to be inseparably attached to their lines of force, so that mechanical attractions and repulsions are exerted between the two magnets.

Experiment 27.—Between the two mutually attracting or repelling magnets interpose successively a number of screens made of different substances: cardboard, wood, glass, brass and iron. Through all but the last the action is transmitted without perceptible change: the iron, however, screens each magnet almost entirely from the influence of the other.

34. **Course of the lines of force in the field of two neighbouring coaxal magnets; representation of the pressures and tensions in the field.**—The mechanical forces described in the last paragraph will be more easily understood if we consider the changes of form experienced by the two superposed fields. In order to see these conveniently, we form the line-of-force diagram in a plane parallel to the axis of two coaxal bar magnets.

For the production of the following line-of-force figures, we may make use of a board about 2 or 3 cm. thick, 20 cm. long, and 15 cm. wide, protected against warping by cross-ribs. In this grooves are to be cut, of such breadth and depth that the magnets to be examined, when fitted into them, may lie flush with the upper face of the board. The grooves are arranged as in fig. 9. In the central longitudinal groove *a a*, the two bar magnets lie coaxally; while in the two grooves b_1 and b_2 they can lie parallel, and in the groove *c* perpendicular to one another. Over all the sheet of paper is laid.

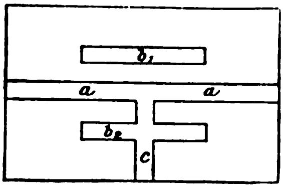

Fig. 9

For the purpose of comparison it will be well once more to form the line-of-force diagram (fig. 4, § 21) corresponding to one only of the two magnets (which are supposed to be as nearly equal in strength as possible). On forming subsequently the line-of-force figures for the field surrounding the two magnets, the extent to which the lines of force diverge in issuing from each of the poles should be compared with what is observed in the case of a single isolated pole.

(a) *Like poles turned towards one another* (fig. 10).—The lines of force proceeding from a pole n_1 of the magnet m_1 are bent back on encountering those which proceed from the neighbouring like pole n_2 of the other magnet m_2. They curve sharply round, and in the more remote parts of the field become more and more nearly parallel to one another. The divergence of the lines of force has thus become much greater than it was in the case of a single isolated pole

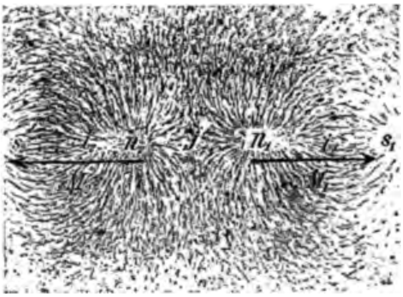

Fig. 10

(fig. 4). There is one place, *J*, in the field which is free from lines of force, and here consequently the resultant force vanishes, the effect due to one pole being neutralised by that due to the other; a short magnetic needle placed in this part of the field would thus be subject to no directing influence, and would behave quite independently of the neighbouring magnetised bodies. The point J is called an indifferent point or 'zero point.'

Sometimes the directing influence of a magnet at a place occupied by a magnetic needle may need to be greatly diminished. This is accomplished, as in the case just discussed, by bringing near the like pole of an auxiliary magnet, which causes a greater divergence of the lines of force. The needle is then said to be astaticised, a plan often adopted in magnetic measuring instruments, in order to increase their sensitiveness.

The deformation of the field common to the two magnets, taken in conjunction with the fact that these magnets repel one another (as indicated in the figure by the arrows), leads us to the conclusion that *the lines of force repel one another perpendicularly to their own direction.*

The significance of the lines of force is chiefly directional, but they also express a certain condition of the medium through which they pass. It may be mentioned here that the condition in question is one in which there is a pressure exerted perpendicularly to the direction of the lines of force. This was the conclusion at which Faraday arrived. (Compare Maxwell's memoir 'On physical lines of force.')

If we assume, therefore, that in the medium transmitting magnetic actions there is a pressure perpendicular to the magnetic lines of force, the line-of-force diagram shows that the two poles n_1 and n_2 will repel one another. Hence the mechanical force observed in experiment 25.

According to the older conception of magnetic phenomena, the seat of the effects was to be sought in the poles, where certain hypothetical fluids, or so-called magnetic masses, were considered to exist. These were supposed to exert upon one another a direct 'action at a distance,' without any intervention of a continuous connecting medium. The medium, however, plays a fundamental part in the phenomena, a fact which experiment 27 helps to illustrate. The most important distinction between the new conception and the old lies in the transference of the seat of magnetic phenomena from the poles to the fields which surround them.

(b) *Unlike poles turned towards one another* (fig. 11).—The lines of force proceeding from a pole n_1 of a magnet m_1 bend round and unite with the lines which terminate at the neighbouring pole s_2 of the other magnet m_2. In the neighbourhood of the axis of the diagram, the lines are only slightly curved, but in the more outlying parts of the field they are more and more spread apart, and pass in wide curves from one pole to the other. Accordingly the lines of force passing from the first pole n_1 to the second pole s_2 are especially closely crowded together. They are less spread out from one another, and much less curved than if either magnet were present alone, and uninfluenced by the other (compare fig. 4). There is a mechanical force urging the two magnets together, as indicated by the arrows. It follows then, that, in addition to the pressure across the lines of force, there must be a tension along them. If the lines of force passing from n_1 to s_2 have a tendency to shorten in their own direction, like stretched rubber cords whose ends are attached to these poles, there will result a mutual attraction, such as we actually observe. The cross-pressure which the separate lines of force exert upon one another keeps these lines from shrinking up, and passing across by the shortest path from n_1 to s_2.

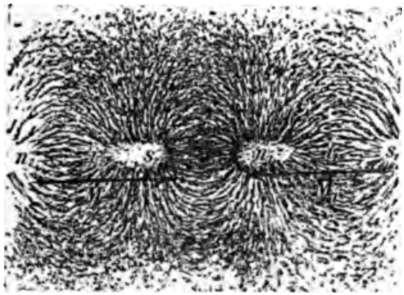

FIG. 11

On the older hypothesis we should simply say that the two poles, endowed with different magnetic fluids, attracted one another at a distance. Faraday did not believe in such centres of attraction-at-a-distance. His description of the phenomenon in terms of the lines of force was much more complete.

We prefer to avoid the terms 'attraction' and 'repulsion' of magnetic poles, directing our attention rather to the state of stress in the medium, the pressure across the lines of force, and the tension along them; for herein lies the essential basis of all our subsequent conceptions.

By supposing all magnetic processes to have their seat in the field, we get rid of a difficulty of conception inherent in the older theory, that effects were produced by something at places where it was not. Action at a distance is now replaced by a continuous transmission of stress through a medium. All media are capable of transmitting magnetic influences, as Faraday proved; that is, they can take up and propagate that type of stress which constitutes the field. We can even avoid, then, introducing into our conceptions another hypothetical substance—the so-called ether. It can be shown that all regions which we are able to investigate are filled with matter. For example, a Torricellian vacuum, above the mercury in a good barometer which has been well boiled out, still contains so much vapour of mercury and glass, that even here we do not need to invoke the aid of any special medium for the transmission of the stresses which constitute the magnetic field.

[But as the transmission of magnetic effects does not become perceptibly less as the number of molecules per unit volume is reduced, we must suppose that the principal transmissive agent is something other than the molecules.]

35. **Mutual action of two long bar magnets placed side by side; lines of force in bipolar magnetic fields.**—We proceed to experiments with two long bar magnets fixed side by side in a vertical position. The action of their extreme ends can be fully examined, and when the bars are long enough we shall be dealing with a magnetic field which is determined wholly by two poles occupying a limited portion of space, and is called a 'bipolar' field.

For the production of figs. 12 and 13 two bar magnets were used, 50 cm. long, 1 cm. thick, and magnetised to the same extent. These could be fixed in a vertical position at different distances from one another in a wooden stand 50 cm. in height and having a base and a cover; the two poles which were turned uppermost being like or unlike as desired.

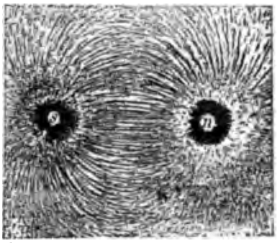

Fig. 12

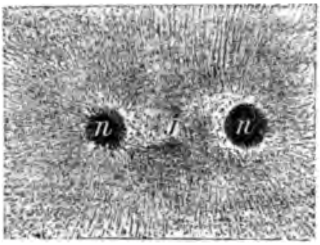

Fig. 13

(a) *Field due to two unlike poles* (fig. 12). The lines of force issuing from the one pole *n* will be absorbed by the unlike pole *s* of the other magnet, as is seen on comparison with the radially spreading lines in a unipolar field (fig. 5). Even the more outlying lines pass round finally to the south pole.

The heaping up of iron filings around *n* and *s*, and the existence of neighbouring bare patches, is due to causes already explained in § 21.

(b) *Field due to two like poles* (fig. 13).—The lines of force proceeding from a pole *n* curve back on encountering those which issue from the corresponding pole *n* of the other magnet. The figure clearly shows how this repulsive force affects the whole system of lines; the chains of filings run everywhere outwards. Between the two poles there is a place J, entirely free from lines of force—a so-called 'zero point.'

Since there is a tension along the lines of force, it follows that the unlike poles of fig. 12 must tend to approach one another; they thus appear to attract one another in a direction perpendicular to the axes of the bars. On the other hand the pressure perpendicular to the lines of force must cause in the like poles of fig. 13 a tendency to recede further from one another, so that there appears to be a repulsive force between them. Here, then, we have the rule of attraction and repulsion between two isolated poles illustrated by means of the iron-filing figures.

We have now, in some measure, arrived at an explanation of the mechanical forces which were described in the first part (*A*) of this chapter. Since a north pole *n* emits lines of force, it will be attracted by a south pole *s* (fig. 12) and if free will move up into contact with it. It will be repelled, however, in the direction of the lines of force by a fixed north-seeking pole N, since the lines of force due to N and *n* encounter one another as they are proceeding in opposite directions (fig. 13). Exactly the reverse of this holds good for the south pole S. Thus if a north-pole is brought into the field of a fixed magnet, it tends to move along the positive direction of the lines of force, while a south pole tends to move in the contrary sense.

36. **Sources and sinks in magnetic fields of force.**—In describing the diagrams of lines of force, we have repeatedly used expressions which are borrowed from the subject of fluid motion. We have spoken of a flow of lines of force, and have referred to such lines as issuing from the north pole and entering the south pole. We must now deal more rigorously with the very important conception of a flux of the lines of force, or, more shortly, flux of force, to which this nomenclature corresponds. The expressions in question are only used figuratively, and in this light they were regarded by Faraday and Maxwell, with whom they originated. We do not assume that there is really anything which flows along the lines of force. The analogy, however, is very remarkable, as a glance at fig. 12 will show. The distribution of lines of force through the field is exactly similar to that of the stream-lines of a fluid, when *n* is the extremity of a tube emitting fluid, and *s* the extremity of another tube through which fluid is carried off at an equal rate. The point *n*, from which lines of force emerge into

the field, is called a source of lines of force, or simply a 'source.' The point *s*, where the lines of force leave the field and enter the magnet, is called a 'sink.' In accordance with § 28 *we are to regard every north-seeking-pole as a source, and every south-seeking pole as a sink.*

These convenient and suggestive terms are due to Maxwell. They furnish us with a purely kinematical analogue for the polar properties of a magnet.

Apparatus for representing the sources and sinks in a magnetic field (fig. 14).—A shallow circular trough A, about 40 cm. in diameter, provided with a rim, and having a conical base, is fitted

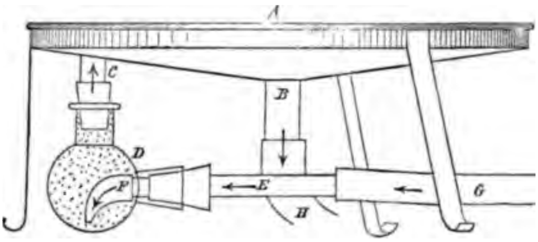

FIG. 14.

with two tubes, B (diameter 3 cm.) and C (diameter 2·5 cm.). The trough stands on three legs, made of strips of sheet metal. A receiver D is attached to C by means of a piece of rubber tube, and into the second tubulure of D a wide glass tube E is inserted through a rubber stopper. On the inner end of E within the receiver is fitted a piece of rubber tube, constricted at the further end, which is bent downwards. The receiver is half filled with sawdust.

If a strong stream of water is now allowed to enter the receiver through G, E, F, it will carry with it some of the sawdust (which when wet through tends to collect at the bottom of the receiver). The sawdust will thus be carried through the tube C into the trough A, and will enable us to follow quite clearly the course of the separate stream-lines, which spread out from the 'source' C (much as the lines of force spread out from the north pole *n*, fig. 12), and curve round till they all converge to the 'sink' B, which they enter, as the lines of force enter the south pole *s*. The water passing down through B is led away by the rubber tube H.

37. The field between two parallel bar magnets.—Let us investigate the field due to two bar magnets, placed side by side upon a horizontal plane with their axes parallel. In this case also it may be seen that the pressures and tensions in the field are sufficient to account for the observed mechanical action.

Experiment 28.—Lay two circular-cylindrical bar magnets side by side on a smooth plane surface; they will roll together if the poles nearest to one another are unlike, while in the contrary case they will roll away from one another.

The line-of-force diagrams. (*a*) *Unlike poles adjacent to one another* (fig. 15).—The lines of force proceeding from the poles n_1, s_1 of the one magnet bend round till they reach the poles s_2, n_2 of the other, and unite with the lines proceeding from these latter poles, thus forming a connection between the two bars, the lines of force in the neighbourhood of the poles being nearly straight. Outside the space between the magnets, the lines of force are bowed out more and more widely, owing to the pressure perpendicular to their direction. The figure shows that there will be a considerable force of attraction urging the two magnets together. This force is due not only to the poles at the extremities of the magnets, but also to parts lying nearer to the middles of the bars, as may be seen from the sparsely distributed lines of force which thread their way across the intervening space. The only place, J, which is free from lines of force is where the lines from the indifferent zones i_1 and i_2 become parallel to one another. If we consider the distribution of the lines of force in three dimensions, as given by fig. 15 in

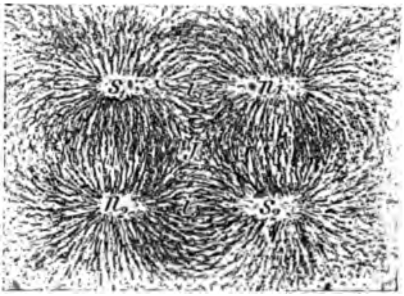

Fig. 15

combination with fig. 5, we see that the positions where the influences from the four sides neutralise one another all lie on a straight line through J, perpendicular to the plane of the paper. A large proportion of the lines of force lie in the space between the two bars; a comparison of their distribution here with that observed in the case of a single bar magnet (fig. 4) shows how they have become concentrated within this space. Even the lines which pass from the ends of the magnets to the outer parts of the field are much less bowed out, and lie much nearer to the middle of the diagram.

(*b*) *Like poles adjacent to one another* (fig. 16).—The lines of force proceeding from the poles n_1 and s_1 of the one magnet curve back on encountering those from the poles n_2 and s_2 of the other magnet. They largely avoid the space between the two magnets, running nearly parallel to one another along the middle line. Between each pair of like poles there will be an indifferent point (J', J''). Owing to the pressure across the direction of the lines of force, the bars tend to move further apart. The same pressure drives the lines of force away from the space between the bars, and causes those in the outer parts of the field to be bowed out more widely.

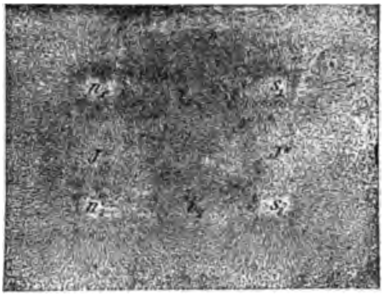

Fig. 16

It may perhaps seem possible that the magnetic effects at a given place might become appreciable, through the lines of force being propagated along chains of iron filings, extending as far as the position in question. This, however, is not the case; on the contrary the magnetic force has at each point a determinate magnitude and a determinate direction in which the filings set themselves. The field will indeed be modified by the presence of the soft iron particles, but only to a very slight extent. Fig. 17 shows again the same disposition of two bar magnets, with like poles adjacent to one another, just as they were used for the production of fig. 16. The lines of force, however, have not been developed (by tapping the paper) as far as the indifferent points *J′* and *J.″* But it will be seen, especially on comparison with the diagram corresponding to a single bar magnet, how the lines of force have been driven away from the space between the two bars. Thus the stresses are seen to be present, even when our chains of filings do not extend to the points *J,′ J.″*

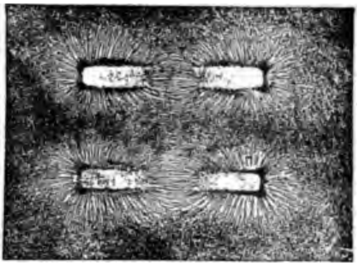
Fig. 17

38. **Transverse effects; rotatory displacements.**—We must now examine the disposition of the lines of force when two bar magnets are laid in the grooves *a* and *c* of the board (fig. 9) so as to have their axes at right angles to one another, an extremity of one being not far removed from the middle of the other. The line-of-force diagram in a horizontal plane (*i.e.* in a plane parallel to the axes of the magnets) is shown in fig. 18.

The lines of force proceeding from the pole n_2 of the magnet m_2 pass mostly to the unlike pole s_1 of the other magnet. From the pole n_1 of this magnet they are driven away by the issuing lines of force which they encounter. Between n_1 and n_2 there is an indifferent point, where the magnetic force vanishes. Under the influence of the pressures and tensions in the field, the bar m_2 in the figure is urged towards the right, as indicated by the arrow. On the right hand, the lines of force with both ends attached to the magnet are striving to become shorter, and thus exerting a pull; while on the left hand a thrust is exerted by mutual repulsion between the lines of force, which encounter and turn away from one another. At the left side there is an indifferent zone, or region of small magnetic force, extending obliquely upwards towards the right.

Experiment 29.—*The mechanical forces.*—Over one end of one of the magnets let a tightly-fitting collar be passed, the collar being provided with a hook so that the magnet may be able to hang vertically from a suspending thread. This magnet, on bringing near to it the middle of another magnet held in a horizontal position, will be attracted towards one pole, in accordance with the course followed by the lines of force.

Experiment 30.—One of the magnets m_1, being supported at its middle upon a needle-point, or hung by a thread so as to turn freely in a horizontal plane, will come to rest pointing north and south. Now bring near the other magnet m_2.

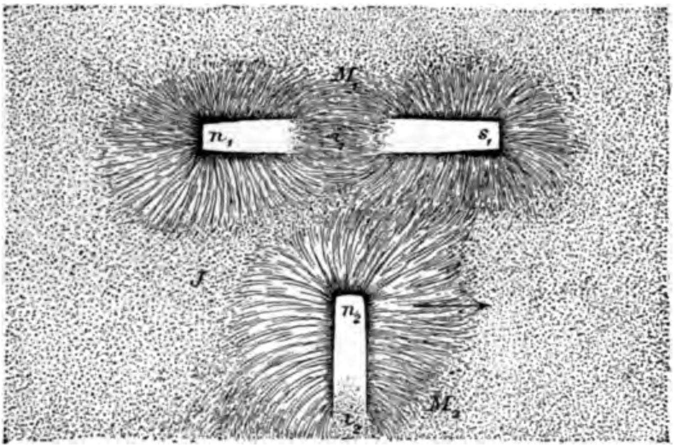

FIG. 18

(*a*) From the east or the west with one or other pole foremost, its axis being always kept perpendicular to the position of rest of m_1, and passing through the axis about which m_1 rotates. *First theorem* (GAUSS and WEBER) : a rotation takes place. This is easily explained by reference to the actions between the ends of the two magnets.

(*b*) From the south or the north, with its axis perpendicular to the position of rest of m_1, which should pass through the middle of m_2. *Second theorem*: rotations also take place. They are less (by about half) than in the previous case (*a*), for the same distance between the centres of the magnets, and as before their sense is determined by the disposition of the poles.

These rotatory displacements are not to be referred to the mutual action of separate point-poles; for, as fig. 18 shows, it is not only the extreme ends of the magnets that take part in this effect, but nearly all parts of the bars, provided we do not restrict ourselves to the employment of very long and thin bar magnets, whose distance apart is considerable.

CHAPTER III

MAGNETISM AS A TERRESTRIAL AND ASTRONOMICAL PHENOMENON

The lodestone is almost the only natural body which is found to have magnetic properties, but these in turn can be communicated to pieces of steel by rubbing with the lodestone. It might seem as if magnetic effects were restricted to a very narrow range; and so, before attempting to enter further into the essential nature of the phenomena, we shall show that they are of far greater interest and importance than our first observations would lead us to suppose. The earth itself is a magnet, and we live in its magnetic field. This field is not constant, but is subject to periodic variations, is convulsed by sudden changes of conditions, and is slowly and progressively altering its distribution. In the variations of the field we can recognise the influence of astronomical events, so that there would seem to be magnetic lines of force connecting our planet with other worlds. The knowledge of this fact gives to magnetic forces a universal interest and significance.

A.—Magnetism as a terrestrial phenomenon

39. **The magnetic elements.**—We have already mentioned (Chap. I., *D*) the north and south direction of a magnetic needle at rest, and as an effect of this kind is to be observed at nearly all places on the earth's surface, we must attribute it to some property of the planet, to some magnetic force which the earth as such calls into play. So far, we have only made use of this terrestrial magnetism for determining

and specifying the polarity of our magnets, but it is now time to investigate more closely the course of the lines of force in terrestrial magnetic fields, and the data needed for their determination. For this purpose we make use of a magnetic needle freely movable in all directions.

Experiment 31.—Let a good unmagnetised steel needle be suspended in such a way that the weights of the two parts on either side of the point of suspension balance one another. Then magnetise the needle by stroking it with another magnet, and having duly observed its polarity, let it turn freely in a place far enough removed from all artificial magnetic fields. One end of the needle is found to dip downwards—in our part of the world, the north-seeking end—the angle of dip being determinate, and the axis of the needle lying in a determinate vertical plane. If we take the needle to different parts of the room, keeping it sufficiently far from considerable masses of iron, such as stoves, etc., the various positions taken up by the needle are parallel to one another, so that the field is sensibly uniform throughout the space which we have examined.

(*a*) *Direction of the lines of force in the earth's magnetic field.*—Our freely movable needle sets itself in the direction of the lines of force, for the complete specification of which direction we require to know two angular magnitudes.

Experiment 32.—Let the needle be suspended over a horizontal plane, on which a north-and-south line (geographical meridian) has been marked out,[1] while a sheet of paper supported in a vertical plane is to be brought quite close to the needle, so as to be parallel to its length. The vertical plane thus determined has roughly, but by no means accurately, a north-and-south direction; it makes a certain angle with the meridian plane—in Central Europe about 10° to 14°, the north-seeking end of the needle pointing west of the true north. This angle is called the magnetic *declination*, and the direction of the vertical plane through the axis of the needle the *magnetic meridian* at the place of observation.

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[1] The meridian may be found with sufficient accuracy for our present purpose as follows: a pin, which is stuck in an upright position into a horizontal plane, has attached to it a card through which a small hole has been bored. About the point vertically beneath the hole as centre, a number of circles are drawn upon the horizontal plane, and marks are made at those points where the circles are intersected by the path of the sun's image, as it moves over the plane. On any given day, each circle will be intersected twice, once before noon, and once after, and the line drawn from the centre of the circles so as to bisect the corresponding arc is the meridian line required.

If we further mark upon the vertical sheet the direction of the needle, and, drawing also a horizontal line, measure the angle between the two (with a protractor, for example), the value obtained will be in central Europe about 60°–68°. This angle is called the *inclination* or *dip*.

The direction of the lines of force in the earth's magnetic field is determined at each place by the values of these two angles, the declination and the dip.

(*b*) *Sense of the lines of force.*—Since a freely turning magnetic needle, whose axis is a line drawn from the south-seeking to the north-seeking pole, always sets itself in the direction of the lines of force, we see that in central Europe the lines of force of the earth's magnetic field deviate by about 60° to 68° in a downward direction from the horizontal, and have also a slight westerly deviation from the south-to-north direction.

We must therefore suppose that somewhere in the neighbourhood of the geographical north pole there is a 'south-seeking' magnetic pole; otherwise our convention, which takes the north-pointing end of the magnetic needle to be a source of lines of force, would be in contradiction to the simple rule of attraction and repulsion between magnetic poles.

(*c*) *Density of distribution of the lines of force.*—For the complete specification of a magnetic field, we must further know how many lines of force per unit area intersect a surface perpendicular to their direction. This number furnishes a measure of the magnitude of the force or intensity of the field, and in this chapter methods will be described which enable us to determine it with accuracy. For the present it may suffice to mention that those methods, applied to the earth's field in this part of the world, give, roughly speaking, one line of force to two square centimetres.

The three quantities, declination, dip, and intensity (or 'total force'), are sufficient to determine completely the magnetic field; they are called the 'magnetic elements' for the place of observation.

40. Model of the lines of force in the earth's magnetic field.—It is important to obtain as definite a conception as possible of the course and disposition of the lines of force in the earth's magnetic field, which plays a fundamental part in certain induction phenomena to be described later. The following model may serve to make the question clearer.

On a foot F (fig. 19) is mounted a brass upright H, which carries a circular wooden disc 10 cm. in diameter and 1 cm. thick, with an adjustment which enables the disc to be set at any desired inclination to the horizon, a semicircle G graduated in degrees being attached to the disc, and turning along with it. The index Z, attached to H, gives the inclination of the disc to the vertical. One side *n* of the disc is painted red, the other side *s* blue. Through the disc, in the direction from *s* to *n*, thirty-nine knitting-needles have been driven perpendicularly, their positive ends (those on the side *n*) being furnished with arrow-heads. The spacing between the needles is as uniform as possible, the points where they enter the disc being the centres and angular points of regular hexagons. Since the area of the disc is about 78 sq. cm., there will be one needle through each 2 sq. cm. Let the model be turned about its base until the needles are parallel to the magnetic meridian, the red side of the disc being towards the geographical north, and then let the disc be tilted until it makes with the vertical an angle equal to the angle of dip, the positive ends of the needles

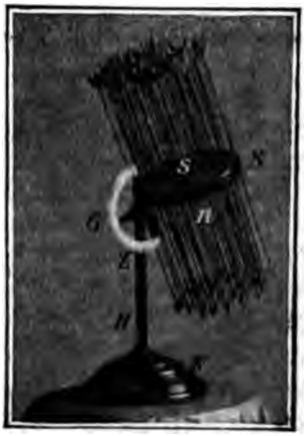

FIG. 19

pointing obliquely downwards. The angle made by the needles with the horizon will also be equal to the dip, and the model will then represent the uniform magnetic field of the earth.

41. **Horizontal component of the earth's magnetic field.**—The needle described in § 39 is not altogether satisfactory for investigating the magnetic force due to the earth, chiefly owing to the uncertainty of the suspension. It is better to use a magnetic needle with only one degree of freedom—that is, freedom to rotate about a vertical axis. To avoid the dipping down of the north end of the needle, arising from the inclination of the magnetic force to the horizon, either the south end must be somewhat loaded, or the point of suspension must be placed somewhat nearer to the north end. Of the total magnetic force, which we will call T, only that component H, whose direction is horizontal, acts on the needle. If i denotes the inclination or dip, the relation between T and H is given by

$$H = T \cos . i \qquad (1)$$

Thus if one of these two quantities be given, the other can be immediately deduced, provided the dip is known. The value of the horizontal component H is therefore an important datum in relation to the earth's magnetic field.

Experiment 33.—A small strongly magnetised steel bar is laid upon a flat cork, which floats in a vessel of still water. The magnet, when at rest, points north and south, but it has no tendency to move bodily towards one side or the other of the vessel. The north-seeking end of the needle is attracted towards the north, the south-seeking end towards the south; and the experiment shows that these two attractions are just equal to one another.

The first compass was of this kind: a magnet upon a floating cork was used very early by the Chinese to guide them in their wanderings through the Steppes and Central Asia.

Two forces of equal magnitude acting upon a body in directions which are parallel, but in contrary senses, are said to constitute a couple. The forces exerted by the earth's field upon a magnet are thus seen to be reducible to a couple.

Model of the couple.—The bar *m* (fig. 20), which represents the magnet, can turn freely about a metal point which projects from the block K. Hooks *h h* are fastened to the ends *n s* of the bar, and, by means of threads *f f* attached to these, equal forces are exerted on the bar by the weights *g g*. The threads pass over pulleys *r r* which are attached to blocks loaded with lead. The two blocks are so arranged that the threads *f f* run parallel to the magnetic meridian, k_1 to the south, and k_2 to the north. The weights *g g* represent the two forces acting on *m*, due to the earth's horizontal component of magnetic force. If the bar is disturbed from its position of rest, the two forces exerted upon it by the threads will bring it back into the meridian plane; and if it is removed from the block K it does not move as a whole either to the south or to the north. We shall make use of this simple model in Section II.

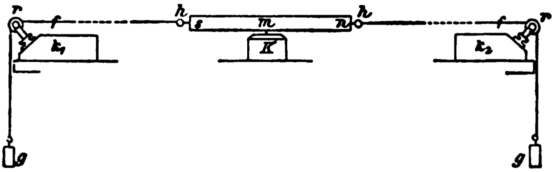

FIG. 20

42. **Declination instruments.**—These consist of two parts, the magnetic needle, which indicates the direction of the magnetic meridian, and some arrangement for determining the geographical meridian. In the simplest form the observer looks through a sight at a fixed mark which has been found by astronomical methods to be due north or due south of the point of observation. In more accurate instruments, a telescope with cross-wires is employed, and if the telescope is mounted on graduated circles (altitude and azimuth), the geographical meridian may be determined by direct astronomical observation (*e.g.* by corresponding altitudes). The line joining the poles does not always co-

incide with that joining the extremities of the needle, and since it is only the latter line that can be directly observed (the positions of the poles being only approximately known) we have here a source of error, against which due precaution must be taken. The error in question will be eliminated if we turn the needle through two right angles about its own axis, and determine the positions of its extremities in the two positions. The mean of the two directions thus found is the true direction of the magnetic meridian.

This can easily be made clear by means of a model. A piece of metal sheet is cut out in the form of an elongated rhombus to represent the needle, two holes being bored at points not on the longer diagonal to represent the poles. Two pins fixed in a board can be made to protrude through the holes, and the line joining these two pins corresponds to the direction of the meridian. The rhombus is to be so laid upon the board that the pins pass through the holes, and then taken up, turned upside down and replaced, so that each pin passes through the same hole as before. Whatever be the deviation of the diagonal of the rhombus from the line joining the pins in the one position, there will be an equal and opposite deviation in the other position.

There is generally a graduated circle just beneath the points of the needle, and if the reading on this circle corresponding to the geographical meridian is known, the declination is the difference between this and the mean of the readings obtained from the ends of the needle.

More accurate values are obtained when a suspended bar-magnet is made to carry a converging lens at one end, and at the other end a pair of cross-wires, in a plane perpendicular to the axis of the bar, and at a distance from the lens equal to its focal length. On placing a telescope so as to sight through the lens from a little distance, the cross-wires may be distinctly focussed. If the observing telescope is provided with another pair of cross-wires, and can move over a graduated circle, the direction of the axis of the bar can be very accurately read. Since the magnetic axis does not in general coincide with the optic axis (the line joining the intersection of the cross-wires to the optic centre of the lens), the bar must in this case also be reversed in position, along with its attachments. Another method is to magnetise a steel tube longitudinally, and to fix at one end a lens of focal length equal to the length of the tube, at the other end a pair of cross-wires. The magnetic meridian is then determined by suspending this tube successively from two diametrically opposite points on the circle bisecting its length.

43. **The compass.**—If the declination is known for any given place, we may conversely use the magnetic needle to determine the direction of the geographical meridian, and hence the four 'points of the compass.' A form of declination-needle convenient for this purpose is the so-called compass-needle. The needle moves over a circle divided into 360 degrees or (for miners) into twenty-four hours or (for seamen) into thirty-two points. The points corresponding to north, south, east, and west are generally marked more conspicuously than the rest. The instrument is enclosed in a case, and is furnished with a contrivance, called an arrestment, which serves to lift the needle off the point when not in use.

To the Italian GIOVANNI GIOJA is due the device of attaching the needle to a divided circle, or a disc marked with the thirty-two points, thus producing the compass so indispensable at sea. The needle (or system of needles), with the disc attached, turns upon a fixed point within a case, which is closed by a glass plate. A fixed mark within the case gives the direction of the ship's head. Since the graduated disc is always maintained in the proper orientation by the needle attached to it, the steersman can read off immediately on the graduations the direction of the ship's head, that is, the direction in which he is steering. The compass usually hangs in a system of movable rings (CARDANI's suspension), and its graduated disc floats upon alcohol, so as to be as little influenced as possible by the oscillations of the ship.

44. **The dip circle.**—A magnetic needle is mounted so that it can turn freely about a horizontal axis passing

through its centre of gravity. The axis is that of a thin steel bar with pointed ends working in jewelled bearings. If the vertical plane in which the needle turns be brought into coincidence with the magnetic meridian plane, the needle points obliquely downwards along the direction of the lines of force. A vertical divided circle enables the positions of the ends of the needle to be read off. The zero reading is set by means of a level or a plumb-line. To determine the dip accurately, it is necessary to reverse the needle, and take the mean of the readings obtained in the two positions.

45. **Equivalence of the earth's field to the field of an artificial magnet; astatic arrangements.**—If the earth is really quite comparable to a magnet, the lines of force in the terrestrial magnetic field must be directly influenced by an ordinary permanent magnet, just as the lines due to another magnet would be.

Experiment 34.—A large declination needle is suspended over a line which gives the direction of the magnetic meridian. At some height above, a large bar magnet is hung in a horizontal position by a strong untwisted thread which passes over a pulley, the centre of the bar magnet being vertically over that of the declination needle. Both magnets are dominated by the earth's magnetic field, so that their red ends point towards the north, their blue ends towards the south. The bar magnet is now gradually lowered; the forces tending to bring back the needle into the magnetic meridian become weaker and weaker, till finally the needle appears to be no longer controlled by the earth's magnetic field, and turns right round. *The effect of the earth's magnetism is neutralised by the influence of the bar magnet.*

The disposition of the lines of force is similar to that of fig. 16. The lines proceeding from the declination needle unite with those of the earth's field, but as the strong and same-way directed bar magnet is brought closer, these lines are more and more driven away, till finally they become so strongly curved downwards that they cease to unite with the lines of the earth's field, and become associated with those of the bar magnet; then the needle turns round end for end.

It is very often useful to partially compensate the earth's field at the place occupied by a needle, and to effect this it is only necessary to place an auxiliary magnet in the magnetic meridian at a suitable distance. This procedure is called *astaticising* the needle in the earth's field, or rendering it *astatic.*

Another convenient method is to fasten together two magnets with their axes running in opposite directions, so as to form an astatic pair of needles. If the needles are nearly equal in magnetic strength, the controlling effect due to the earth's magnetic field will be very small.

46. **Values of the magnetic elements at different places; magnetic charts.**—If we make determinations of the declination, dip, and total magnetic force at various points of the earth's surface, we find different values from which the distribution may be deduced in accordance with definite laws. Generally speaking the further we go north or south from the equator the more markedly does the magnetic needle point downwards, the dip increasing with the latitude. In the northern hemisphere it is the north-seeking pole that points downwards; in the southern hemisphere it is the south-seeking pole. In the neighbourhood of the equator the earth is encircled by a line at each point of which the needle sets itself horizontally, so that the angle of dip is zero. This line is called the 'magnetic equator,' and is not coincident with the geographical equator. There are also points, one in the northern and one in the southern hemisphere, where the direction of the dipping-needle is exactly vertical. From the analogy to artificial magnets, these are called the 'magnetic poles of the earth.' They do not coincide with the geographical poles. The north magnetic pole is in 70·5° north lat. and 98·5° west long. from Greenwich, a point in the North American Archipelago; the south magnetic pole is about in 74° south lat. and 148° east long. The line joining these poles does not pass exactly through the centre of the earth.

In order to realise more clearly the disposition of the lines of force, we may make use of a small globe of papier-mâché, within which is placed a bar magnet in the direction of the magnetic axis of the earth. To explore the field, we may use as before a piece of strongly magnetised steel wire, suspended from its middle point by a thin cocoon fibre, so as to have freedom to turn in all directions. If the enclosed magnet is sufficiently strong, the disturbance introduced by the influence of the earth's magnetism will be but small, the needle everywhere setting itself in the direction of the lines of force which proceed from the globe.

If we draw lines passing through all points for which the angle of dip has the same value, we obtain curves whose general disposition is not unlike that of the parallels of latitude. They do not, however, coincide with these latter, nor are they parallel to one another. They are called 'isoclinic lines.' The isoclinic line for a dip of 0° is thus the magnetic equator. If lines are drawn joining the points of equal declination on the earth's surface, the so-called 'isogonic' lines are obtained. They are of great importance in navigation, since they render it possible to take account of the declination of the compass at each place. But there are other lines which have a greater interest for us, as indicating the distribution of terrestrial magnetic force; if we follow always the direction in which the declination needle points, either to the north or to the south, we shall necessarily arrive at a magnetic pole. (It is only at the poles themselves that the declination needle fails to take up any determinate position in azimuth.) In this way we obtain curves which give at each point the direction of the horizontal component of the earth's magnetic field, thus determining the magnetic meridian, which does not in general coincide with the geographical meridian.

The intensity of the earth's field is least in the neighbourhood of the equator, and becomes greater towards the poles, though not according to any regular law. The places where the magnetic force is a maximum are not situated at the poles, but at some distance from them. The earth is thus seen to be an irregularly magnetised body. The variations of intensity or 'total force' may be indicated by means of curves such that for all points on any one curve the 'total force' is the same. These are called *isodynamic lines.*

47. **Disposition of the lines of force in the space surrounding the earth.**—The magnetic field of the earth comprises all space surrounding our planet, so far as we can detect in it any magnetic influence. The portion which we are able to explore is necessarily confined to a very thin shell over the earth's surface. Even when the magnetic needle is used by the miner, or is taken up a mountain or in a balloon, the range of our direct observation is very small compared with the whole volume of the earth. It is apparently only the *aurora borealis* which can furnish us with any certain information as to the course of the lines of force in the higher regions of our atmosphere; but we can obtain an approximate conception of them when we know the direction in which the dipping-needle sets.

We must picture the lines of force as emerging with considerable density in Victoria, South Australia. At greater distances from the south pole (magnetic north-seeking pole), lines of force also emerge from the earth, but less thickly, and with a component of direction towards the north. Still further northwards we come to the indifferent zone of the terrestrial magnet, which is a somewhat broad band running equatorially round the earth. After this there are regions where the lines of force from without re-enter the earth, somewhat few at first, and nearly horizontal in direction, but always denser and more nearly vertical as we approach the north magnetic pole, which has the polarity we have called south-seeking. Finally when we reach this point, in the North American Archipelago, the lines of force have something like their greatest density of distribution, and run vertically downwards into the body of the earth.

The disposition of the lines of force along a magnetic meridian may be illustrated by means of a magnetised sphere, for the production of which we must employ an electric current, a phenomenon to be described later. On the two halves of a soft iron sphere K, about 2 cm. in diameter (fig. 21) V-shaped channels are cut, so that a covered copper wire 1 mm. in diameter can be wound on; in the figure, only the cross-sections of the windings are shown. These are supposed to run from left to right—that is, in the sense of the corresponding numbers. The ends of the wire are twisted together and are led off from the middle zone (for example, from the centre of the figure, either above or below). Two brass hemispheres are fitted over the whole arrangement, having only small notches cut in the rim to allow the conducting wires to pass through.

The brass sphere with its contents represents the earth, the iron sphere representing the interior of the earth (the high mean density of the earth shows that heavy metals enter largely into its structure, and amongst these iron is probably widely distributed). The brass covering itself represents the region accessible to us: rock, water, and air. On sending a current through the windings, the sphere becomes magnetised.[1] Now take a sheet of paper in which a circular hole is cut, large enough for the sphere to pass through, and arrange it horizontally so as to coincide with a diametral plane of the sphere. On forming the line-of-force diagram by means of iron filings, we obtain a very characteristic figure.

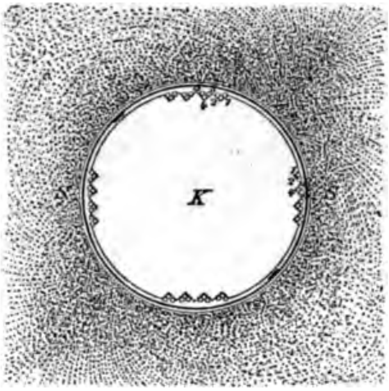

FIG. 21

Fig. 22 shows the line-of-force diagram in a plane through the axis of symmetry (perpendicular to the windings), *i.e.* in a magnetic meridian plane of the sphere. The white circular patch in the middle is the space occupied by the sphere K, in place of which we may imagine the earth. The short lines drawn obliquely inwards from the circumference indicate the direction of the earth's axis. If the figure is to be projected, the sphere must be placed within a circular aperture (about 2 cm. in diameter) in a glass plate, which is held horizontally, and so as to coincide with a magnetic meridian plane; the conducting wires, which pass away to one side, produce no sensible disturbance of the field. We shall afterwards have occasion to speak of the resemblance between these figures and the corona of the sun.

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[1] A few ordinary cells are sufficient for the production of the current.

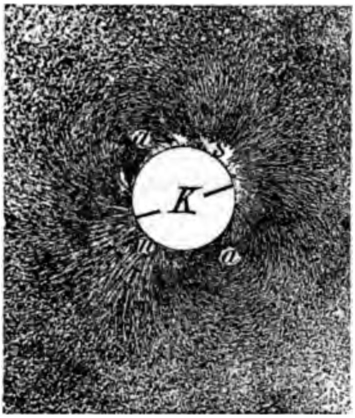

FIG. 22

48. **Variations in the magnetic condition of the earth.**—The system of lines of force is not invariably connected with the earth, but undergoes continuous changes. These changes affect all three 'magnetic elements' at any given place, as well as the distribution of magnetic force all over the earth's surface. The following variations have been distinguished:

(*a*) *Periodic variations*, which are repeated again and again after the lapse of equal intervals of time. Some variations of this kind are related to the earth's rotation about its axis; they have a diurnal period. The north-seeking end of the declination needle, in mean latitudes of the northern hemisphere, has an easterly displacement at about eight o'clock in the morning, then as the morning advances it slowly moves towards the west, attaining its greatest westward displacement between one and two o'clock in the afternoon, and then moving back towards its most easterly position, which is once more reached at eight in the morning, the movement being slowest during the night. The declination needle in the southern hemisphere experiences movements exactly opposite to these. At about nine in the morning the north-seeking end of the needle has its greatest westerly displacement, and it then moves eastwards until two or three in the afternoon. At the same time the dipping-needle shows corresponding periodic changes; the dip increasing during the morning, and diminishing during the afternoon. A needle quite free to turn then would move in such a way that an observer looking along the positive direction of the axis (from south-seeking to north-seeking pole) would see the north-seeking pole describe a small closed curve in the sense in which clock hands move. The direction of the magnetic force through a given fixed point will accordingly describe a conical surface in the same sense. The deviations of the needle from its mean position are small, the angular displacement just described having an amplitude of only a few minutes. At the same time there are small changes in the density with which the lines of force are distributed, that is, the intensity is variable.

The changes in the earth's field have further an annual period. In both hemispheres, the annual variation consists in an eastward displacement of the declination needle through a few tenths of a minute of arc when the sun is north of the equator (*i.e.* in our summer), and a westward displacement when the sun is in the southern hemisphere (in our winter). In both hemispheres, also, the dipping-needle shows an increase in the angle of dip during December and January, and a decrease from June to August. At all places on the earth's surface the 'total force' is greater from October to March than during the est of the year.

(*b*) *Secular variations.*—The whole system of lines of force, and consequently all the magnetic curves which serve to characterise the earth's field, are undergoing a slow progressive change; but it has not so far been possible to determine whether this change will ultimately be reversed, so as to restore the former condition of things. It may be that the variations in question are periodic, the periods embracing hundreds of years.

For example, the system of isogonic lines for Europe is moving slowly towards the west, the isogonics corresponding to higher values of the declination moving further out into the Atlantic Ocean, the westerly declination becoming slowly less in our part of the world (about six or seven minutes of arc in the course of a year). The magnetic equator is continuously shifting from east to west, and with it the whole system of isoclinic lines, the angle of dip throughout Europe gradually decreasing. The total magnetic force or intensity at any given place is also changing; at some places it is becoming greater, at others it is becoming less.

(*c*) *Transitory or sudden variations.*—These occur in response to no apparent cause, and produce 'disturbances' of the whole system of lines of force. Such disturbances often affect large portions of the earth's surface. Though they may in many cases be produced by earthquakes and other terrestrial events, they are also directly influenced by extra-terrestrial causes, and will accordingly be considered under *B*.

49. **Luminous phenomena of the poles.**—Very closely connected with the earth's system of magnetic lines of force are the luminous phenomena which at certain times produce an illumination of the upper layers of the atmosphere in both polar regions, often on a very magnificent scale. They are a direct visible expression of the magnetic condition of the earth, and are so especially related to the variations of this condition that they were justly called by HUMBOLDT 'magnetic tempests' (*magnetische Ungewitter*). When a magnetic storm passes over the earth, and the magnetic needles are set trembling, the lights at the northern and

southern poles, at no time quite extinguished, become especially brilliant.

The luminous phenomena are very varied in form, and may be classed under the following heads:

1. The best known form is the so-called 'northern lights' or 'southern lights' (*aurora borealis*, or *australis*), and consists of luminous streamers, whose light varies greatly in colour, from red and orange on the one hand to yellow or green on the other. These streamers flicker unsteadily to and fro, passing downwards from the higher and more rarefied regions of the atmosphere, and *vice versa*.

2. Numerous and careful observations have established the fact that the streamers or rays of the aurora are everywhere nearly coincident in direction with the magnetic lines of force. The separate rays have therefore the form shown in fig. 22.

3. This luminosity along the lines of force does not, however, occur everywhere to the same extent. It is strongest of all over a ring, or, according to Nordenskjöld, over two concentric rings, which surround the pole somewhat excentrically. Within these rings, where the lines of force run nearly vertically and lie most thickly together, the rays are absent. They thus surround the magnetic axis of the earth like two candle-rings, whose profile follows the direction of the lines of force.

This disposition of luminescent particles of our atmosphere serves to account for a number of characteristic features of the polar light.

(*a*) *The occurrence of points of convergence.*—If we are within a region where there is luminosity along the lines of force, we see, on all sides of us, rays or streamers reaching up into the sky. Since the lines of force within the range of our observation from any given spot are sensibly parallel to one another, they appear to us to converge to a vanishing point overhead; just as the rails of a long, straight railroad appear to converge towards a distant point. Thus there seems to us to be a point in the sky (the corona) from which the rays diverge.

(*b*) *The luminous bow and the dark segment.*—If we are near the boundary of a region in the highest atmospheric layers of which luminous phenomena are observable, we shall only see that portion of the luminous ring which is nearest to us, appearing as a bright band over the horizon. From this band luminous streamers are given off, but do not converge to a vanishing point (that point of the heavens towards which the south-seeking or north-seeking end of the dipping-needle points). Since the luminous ring, which we have compared to a candle-ring, does not extend right down to the earth, but ends high up in the atmosphere, we are able to see through into the non-luminous inner space, which appears dark by contrast. This explains the especial darkness of the sky which is observed just below the luminous bow, and is called the dark segment.

(*c*) *The streamer form.*—If the luminosity extends over only a few bundles of lines of force, we shall see a number of narrow, parallel stripes, reaching downwards from the sky. If a magnetic storm passes over the earth, the lines of force will wave to and fro like the stalks in a cornfield, the bands of light swelling and bellying out as if blown by the wind; an effect noticed by many observers.

(*d*) *Visibility of the polar light.*—The polar light, as we have already mentioned, is not most frequent over the magnetic poles of the earth, but is chiefly to be noticed in closed curves which surround the magnetic poles at a greater or less distance. If we draw a line through those points for which the relative frequency of the polar light is the same—that is, if we construct the so-called isochasmic lines (FRITZ)—we observe that towards the magnetic pole the visibility gradually increases, but before the pole is reached it very quickly decreases again, so that at the pole itself the luminous phenomena are much rarer than in somewhat lower latitudes. The curve of greatest frequency surrounding the North Pole can be drawn with considerable accuracy; that corresponding to the South Pole has also been constructed, though the data furnished by observation are in this case very scanty.

B.—Magnetism as an extra-terrestrial influence

Magnetic effects of extra-terrestrial origin cannot be directly recognised, but the variable magnetic condition of our earth shows conclusively the part which they play. In particular, the so-called disturbances of the earth's magnetic lines of force clearly point to the influence of other heavenly bodies. Thus magnetism is a universal bond which unites world to world.

50. **Influence of the sun on the magnetic elements of the earth.**—Above all, it is the sun, the central body of our planetary system, which exerts a magnetic influence on the earth. The effect is not a thermal one, which thus indirectly modifies the distribution of the lines of force, but a direct magnetic effect due to the sun itself, for the variations do not correspond to the changes of temperature (which, as we know, lag considerably behind the apparent motion of the sun), but follow the position of the sun without retardation.

There are three phenomena which produce sensible variations in the magnetic elements of the earth's field :

(*a*) *Period of the annual revolution of the earth about the sun.*—When it is winter in our hemisphere, the earth, moving in its elliptical orbit, is nearer to the sun than in summer. At the same time the dip and total force throughout both hemispheres have a somewhat augmented value. Thus, when the earth is nearer to the sun, the lines of force enter and leave the body of the earth more nearly vertically, and are more closely crowded together. In both hemispheres also a part of the variation of the declination-angle follows the annual changes in the relative positions of sun and earth.

(*b*) *Period of the sun's rotation.*—The sun rotates about an axis in the same sense as the earth, the period of a complete rotation being twenty-four and a half days; but since the earth at the same time is travelling round the sun, it is twenty-six days before the sun presents once more the same aspect to the earth. Both the horizontal magnetic force and the daily variation of the declination-needle have a twenty-six-day period, which must correspond to the magnetic action of the sun as a whole.

(*c*) *Periods of solar activity.*—The radiation of heat and light from the sun is not always the same in amount, but varies from time to time like the aspect of his disc. We can distinguish without difficulty between times of greater and less solar activity. The former are characterised by a great abundance of spots, faculæ and protuberances, while in the intermediate periods of comparative quiescence these phenomena are less both in number and in degree. The alternations occur pretty regularly, with a period of eleven years, while the variations of the terrestrial magnetic elements have also a component with an eleven-year period. The diurnal variation of the declination is so closely related to the variations of solar activity that its mean value for any year may be reckoned to a fraction of a minute of arc from the number of sun-spots during the year.

Again, in mean latitudes, the frequency of the *aurora borealis* is related to the same period. At the times of greatest activity, that is when the sun-spots are most numerous, luminous phenomena, similar in character to the *aurora*, are to be observed in the highest strata of the atmosphere at nearly all places on the earth's surface.

The distribution of the earth's magnetic lines of force is influenced not only by solar activity in general, but by individual spots and protuberances. Thus a specially violent eruption on the sun is almost invariably associated with considerable magnetic disturbances (magnetic storms) on the earth. So far, indeed, it has not been found possible to establish a definite correspondence between the individual phases of the solar and terrestrial phenomena; but it is certain that the sun produces magnetic effects which are appreciable here. It does not follow that the sun behaves like an immense permanent magnet rotating about an axis.

51. Magnetic theory of the sun's corona.—The above relations lead us naturally to inquire whether some of the phenomena to be observed in the sun are not to be explained as the result of its magnetic properties. An observer looking at the earth from a distance would be able to infer the course of the magnetic lines of force from the luminous rays of the *aurora*. The sun possesses, in fact, something which is very similar in character and is called the 'corona.' At times of no special solar activity this forms a somewhat thin, faintly luminous envelope, enclosing the ordinarily visible body of the sun, and exhibiting no special features. At times, however, it becomes greatly extended, and acquires a striated structure, appearing as a network of curved rays, some of which projected far into the atmosphere of rarefied gas by which the sun is surrounded. Suppose the line-of-force diagram of the magnetised sphere in fig. 22 to be formed for all planes through the centre of the sphere. The whole system of lines, as seen from a distant point, will form a kind of network, having the closest resemblance to the fully developed corona of the sun. It has been shown experimentally that rarefied gases become luminous more readily when placed in a magnetic field (EBERT), the rarefied atmospheres of the sun and earth being influenced in like manner by the magnetic fields of these bodies. Thus in a certain sense the rays of the corona may be regarded as the sun's magnetic lines of force rendered visible.

52. **Influence of the moon on terrestrial magnetism.**—The moon, which is nearer to us than any other heavenly body, has a considerable influence on the magnetic lines of force of the earth. It gives rise to a periodic movement of the declination-needle, the disturbance exhibiting two maxima and two minima between two successive transits of the moon across the meridian of the place of observation. There is also a lunar variation of the dip and of the horizontal force.

53. **Effect of the planets on terrestrial magnetism.**—A relation has long been recognised between certain variations of the terrestrial magnetic elements and the positions of the nearest planets, Venus and Mars, while recently, according to Leyst, it has been established that all the planets produce magnetic effects upon the earth. Especially re-

markable is the influence of Mercury, the planet which is nearest the sun, and which, unlike all the other planets, is greater in density than the earth.

Venus, like the moon, exhibits changes of phase, so that at periodically recurring times it appears as a narrow crescent, a considerable part of its dark hemisphere being turned towards us. At such times especial luminous phenomena are often observed, and these have been supposed analogous to the *aurora*.

To obtain information on this point, it is important to find whether our observations show a periodicity in the brightness of this phenomenon. If it should prove to be related to the eleven-year periods of solar activity like the polar lights of the planet Earth, the similarity of nature of the two phenomena would be rendered very probable.

CHAPTER IV

THE MEASUREMENT OF FIELDS OF FORCE

IN this and the following two chapters some simple calculations will be introduced. The reader who wishes for the present to confine his attention to qualitative relations may proceed at once to Chapter VI.

A.—Static methods of measuring magnetic magnitudes; the absolute system of units

We shall now consider the nature of the magnitudes which are involved in quantitative magnetic determinations, and we shall describe those methods of measurement which depend on balancing magnetic forces by gravitational forces of known amount.

54. **Gravitational field of force.**—At all places on the earth's surface we find that freely movable masses have a tendency to move downward, this tendency being due to their weight. Since the form of the earth is approximately spherical, this may also be expressed by saying that the weights of all bodies are forces directed towards the centre of the earth, the lines of force of the earth's gravitational field all proceeding radially from this point. They differ, however, from the lines of force of magnetic (and electrical) actions in not being related to two opposite kinds of polarity; the only known gravitational forces are attractions, never repulsions. Every body, in accordance with NEWTON'S law of universal gravitation, exerts a force of attraction on all neighbouring bodies, and the lines of force emanating from such a body run counter to those of the earth, like the

magnetic lines of force from two poles of the same name. The planet, so far as we know it, consists entirely of constituent parts which behave, in relation to gravity, exactly like the body under consideration; so that in the present case there can be no union of lines of force of the kind observed in the case of unlike poles. Nevertheless attraction takes place. If we consider the disposition of the lines of force in the neighbourhood of two like poles (*cf.* fig. 29, Plate I., at the end of the book), we see that along the gravitational lines of force there must be not a tension, but a pressure; or, as MAXWELL says, *vis a tergo.*

We shall have to speak later of the kinematic aspect of this theorem.

The absence of opposite polarities in relation to gravitational attractions may in a certain sense be said to be confined to inorganic bodies. Plants evidently possess some means of overcoming the pressure along those lines of force, which act on the matter of their cellular tissues, so as to experience an upward pull. This is seen in the growth of their stems against the direction in which gravity acts. By the researches known to botanists as Knight's experiments, on the growth of plants on rotating discs, it is proved that there is really some influence acting against gravity, just as matter of opposite polarity might be supposed to do. A plant growing in the gravitational field of the earth may be said to present a certain analogy to a magnetic needle in a uniform magnetic field. If the plant is displaced from its original (vertical) position, root and stem as they grow bend round until the general lie of the axis of the plant is once more in the direction of the lines of force; a result which reminds us of the tendency of a magnetic needle when displaced to return to its equilibrium position.

[The analogy here suggested is not a very close one: and it can hardly be seriously held that there is any evidence of the existence of gravitationally repelled matter in a growing plant. The plant, to all appearance, maintains its upright position very much as a flagstaff does, in virtue of the stiffness of its structure and the fixation of its base in the ground. If the plant be uprooted, and laid upon a hard, horizontal surface, it does not rise up and stand on end, nor does it show any greater preference for the upright position than might be expected from so much inanimate matter. That a plant with a firm hold for its roots always tends to *grow* upright implies that the development of its tissues is in some way responsive to gravitational influence, but suggests no purely dynamical phenomenon of an unfamiliar kind.]

55. Comparison of magnetic force with the force of gravitational attraction.—We shall now offer some observations on the relative magnitudes of the two kinds of forces, gravitational and magnetic, which we have to compare with one another. Without recourse to any special measurements, we know at once that magnetic forces greatly exceed in magnitude the gravitational attraction which all bodies exert upon one another. Two bar magnets with unlike poles in contact may adhere so that the entire weight of one of them is easily supported against gravity, while the same bars, when unmagnetised, exert on one another so small an attraction that ordinary observation would quite fail to discover it, some highly sensitive method, such as weighing against the torsion of a quartz fibre, being needed to render it apparent.

The fact that the weights of bodies at the earth's surface are so considerable is due of course to the great size of our planet. The magnitude of the masses concerned explains also how gravitational attractions, so weak in comparison with other forces, can so greatly affect the motions of the heavenly bodies.

56. Fundamental units: the centimetre-gram-second system.—In regard to physical measurements there is nothing of such importance as our system of units. After long and difficult international labours, a universal system for scientific and technical purposes has been established. The fundamental units are defined with reference to the size and period of rotation of the earth. We can assign numerical values to all the measurable quantities occurring in nature when *three* fundamental units have been chosen:

(1) For *length*, the centimetre (cm.), which is the thousand-millionth part of a quadrant of the earth, measured along a meridian of longitude from equator to pole.

(2) For *time*, the second (sec.), which is the 86,400th part of the mean solar day.

(3) For *mass*, the gram, which is the quantity of matter in one cubic centimetre of distilled water at + 4° centigrade.

From these three units alone without the aid of any further conception we can elaborate the whole system of measurements required in mechanics. For some purposes it is convenient to choose our units differently, the unit of mass being replaced by a unit of energy; energy, like matter, being indestructible.

The term 'absolute' is applied to such measurements as are based on these fundamental units, or on any of the units derived from them. The system is also called the centimetre-gram-second system (C.G.S. system).

57. **Absolute measurement of force: the dyne.**—Every portion of matter within the earth's gravitational field experiences a determinate attraction in the direction of the lines of force (approximately towards the centre of the earth), and if free to move will have a uniform acceleration in this direction. For a given mass, the acceleration is proportional to the force, and the force is further proportional to the mass, so that the force may be conveniently measured by the product of the mass and acceleration. The corresponding definition of the unit of force is the following:

The unit force is that which produces unit acceleration in the unit of mass.

This unit is called a *dyne* (from the Greek δύναμις = force). Since the action of gravity on the unit (or any other) mass produces, in these latitudes, an acceleration of about 981 units, the weight of one gram is about 981 times the unit of force—that is, 981 dynes; while 981 centimetres per second per second is the measure of the intensity of the gravitational field.

Since the weight of a gram is 981 dynes, or roughly speaking 1,000 dynes, we see that the unit of force which we have adopted is but little greater than the weight of a milligram.

Gravity has the unique property of acting equally upon all kinds of matter. While the behaviour of the magnetic lines of force in passing through iron, nickel and cobalt is quite different from what would be observed in the case of bismuth, &c., and the mechanical forces exerted under conditions otherwise quite similar are by no means the same, the 'permeability' to gravitational lines of force is the same in all substances (Bessel). The magnetic permeability further depends on the intensity of the magnetic force itself, while, in the case of gravity, it may be that the excessive smallness of the strength of field to be observed conceals from us the differences between the properties of various substances.

58. **Measurement of magnetic attractions in dynes by means of the balance.**—We shall take, as the simplest example of a force due to magnetic action, the attraction between unlike poles of two similar and equally magnetised steel rods; and this we shall determine in absolute measure, using an ordinary sensitive chemical balance to equilibrate it by the weight of a known number of grams. Since the weight of a gram is 981 dynes, we obtain at once the value in dynes of the force of magnetic attraction. We shall content ourselves with ascertaining the order of magnitude of the force in question.

Experiment 35.—Take two knitting needles, which produce equal effects in deflecting a compass-needle, and may therefore be considered equally magnetised. Suspend one of the needles (needle 1) vertically from one end of the balance, and let the weight required to counterpoise it be P_1 grams. Then approximate to it from below the attracting pole of the other needle (needle 2), which has fastened over its upper end a plate of glass of known thickness. The distance between the attracting poles will be approximately equal to the thickness of the glass; glass having so nearly the same permeability as air that the force of attraction which we measure is sensibly the same as if there had been nothing but air between the poles. After waiting for the balance to come once more to rest, gradually add to the weights in the scalepan, until the force of attraction between the poles is just balanced, and let the total content of the scale be then P_2 grams. The force required is thus $(P_2 - P_1) \times 981$ dynes.

Numerical example: weight of the magnetised needle 1 with its attachment to the balance $P_1=9{\cdot}23$ grams. Greatest weight required to balance the attraction across the glass plate $P_2=9{\cdot}35$ grams. Hence the magnetic attractive force $f_{12}=0{\cdot}12\times 981=$ 118 dynes.

59. **Dimensions of physical quantities; dimensions of force.**—Surfaces are said to be of two dimensions in length, while volumes are three-dimensional. We may express this by writing the number of dimensions as an exponent over a symbol representing the nature of the fundamental unit; thus if [L] stands for some quantity of the nature of a length:

The dimensions of area are $[L]^2$
The dimensions of volume are $[L]^3$

We have corresponding analogous expressions for the dimensions of all derived units, formed by combining the fundamental units in any manner whatever. Thus:

The dimensions of velocity are $[L]/[T]=[L]\,[T]^{-1}$ where [T] stands for some quantity of the nature of an interval of time; and again

The dimensions of acceleration are $[L]\ [T]^{-1}/[T]=$ $[L]\,[T]^2$, because it is a velocity (increase of velocity) divided by an interval of time.

The dimensions of force are $[M]\,[L]\,[T]^{-2}$ where [M] stands for some quantity of the nature of a mass; a force being measurable as the product of a mass and an acceleration.

60. **Decrease of magnetic effect with increase of distance.**—We may easily perceive that the influence which two magnets exert upon one another becomes less when we interpose between them a greater thickness of a medium whose permeability is small; for example, when the magnets are surrounded by air, and we draw their poles further apart. In experiment 35, if we interpose two or more glass plates between the mutually attracting poles, we can measure in dynes the corresponding changes in the force of attraction. If it were possible to obtain a pole from which

the lines of force radiated equally in all directions, the magnetic force due to such a pole at a distant point might be expected to diminish in proportion as the square of the distance increased, for the same number of lines of force are distributed over a greater spherical surface, whose extent is proportional to the square of the radius. Hence the number of lines of force which cut through any given area normal to their direction (for example, the unit of area) varies in inverse proportion to the square of the distance.

It was Coulomb who first established the truth of the law of inverse square by direct measurements upon long thin bar magnets. If the magnetic influence is propagated continuously from place to place through the surrounding medium, it is evident from purely geometrical considerations (fig. 23) that the effect at a place distant r cm. from the centre of attraction or repulsion will be only $1/r^2$ of the effect at a place whose distance is 1 cm. This law holds good for all agents whose influence is exerted equally on all sides, for example, light, sound, thermal radiation ; always supposing that the medium has no absorbing effect on the influence in question.

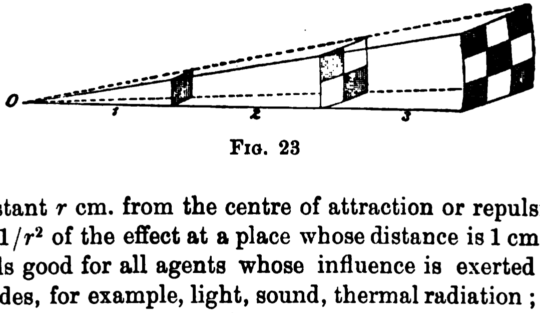

FIG. 23

61. **The Newtonian law of universal attraction.**—Newton had in 1666 assumed the law of variation as the inverse square of the distance, for the gravitational attraction which all bodies exert upon one another, and which constitutes the weight of bodies at the earth's surface. The truth of the assumption was established when it was found to give correctly the motions of the planets round the sun and of the moon round the earth. The planets are nearly spherical in form, and produce the same gravitational effect

as if their entire mass were concentrated at their centres; we may therefore look upon them as equivalent to attracting points, to which the Newtonian law of attraction is directly applicable. Great attention was attracted to this result, owing to its fundamental importance in dynamical astronomy, and it is not surprising that when bodies were found to exert other forces upon one another, such as magnetic forces, a similar law should have been anticipated. But though the planets may be regarded as attracting points, magnetic bodies must be treated somewhat differently. When a magnet is in the form of a long thin bar, the Newtonian law of the inverse square of the distance holds good for the force exerted by each of its poles. It is not to be concluded, however, that the attraction in this case is due to some special magnetic matter, distinct from ordinary substances.

This conclusion, however, was for a long time accepted, the mathematical theory based upon it reaching a very high development even before Coulomb's time and leading to the discovery of many new relations. The hypothesis is now no longer necessary and a little reflection will show its inadequacy. For the Newtonian law takes account of attractions only, and the forces due to magnetic action involve the consideration of two opposite polarities. We should therefore need to formulate some additional hypothesis, such as that of positive and negative magnetic centres exerting actions at a distance which so long prevailed. The forces due to magnetic bodies can only be regarded as arising from point sources of attraction in certain special cases. Such magnetic systems, with poles giving rise to a radial distribution of lines of force, are only of subordinate importance, for example in some technical applications.

62. **Strength of pole.**—The analogy between the mutual action of magnets and the attraction of gravitation may be carried a step further. According to NEWTON's law, the attraction between two bodies depends on their masses as well as on the distance between them. A similar relation is to be assumed in the case where two magnets act upon one another; so that we speak of *quantities* of north-seeking

or south-seeking magnetism, or of magnetic 'masses.' To these modes of expression we owe certain conceptions which are so convenient in the theory of magnetism that we shall retain them in our more modern exposition of the subject. One such conception is that of the *strength of a pole*; it may be deduced from experiment without recourse to the hypothesis of magnetic masses acting upon one another at a distance. If we compare with one another the poles of different magnetised bars, by examining their action on one and the same bar, differences will be observed under quite similar conditions, and these can only be due to differences of strength amongst the poles themselves. We may express this result numerically by associating with each pole a corresponding number.

Experiment 36.—The magnetised needle 1 of experiment 35 is suspended from the balance as before, but in place of the needle 2, equally strongly magnetised, take some other needle 3, of the same size, but magnetised to a greater or a less extent, the force of attraction between 1 and 3 being f_{31}. Let the ratio $f_{21}:f_{31}$ be denoted by n. Now replace the suspended needle by another needle 4, magnetised to any degree, determine its weight, and find in dynes the attractive forces exerted upon it by the corresponding poles of the needles 2 and 3; call these f_{24} and f_{34}. If we now calculate the ratio $f_{24}:f_{34}$ we obtain the same value as before; that is, $f_{24}:f_{34}=n=f_{21}:f_{31}$.

Numerical example.—$f_{21}=118$ dynes, as was found in experiment 35; $f_{31}=283$ dynes; and $283:118=2{\cdot}4$. The needle 4 whose weight P_4 is 11·52 grams being now suspended in place of 1, it was found that $f_{24}=187$ dynes and $f_{34}=449$ dynes; the ratio $f_{24}:f_{34}=449:187=2{\cdot}4$, as before.

The numerical value of the attraction involves two coefficients, each of which is a characteristic of one pole alone, and independent of the other pole. In other words, the mutual attraction (or repulsion) of two poles is proportional to the product of two numbers, each of which measures quantitatively the effect of one pole, and is called the *strength* of that pole.

If the strength of the poles used in the last experiment are denoted by m_1, m_2, m_3, m_4, f_{21} is proportional to $m_2.m_1$ or, if we please, equal to km_2m_1 where k is a constant quantity. In the same way $f_{31}=km_3m_1$, so that $n=m_2 : m_3$. Again if $f_{24}=k'm_2m_4$ and $f_{34}=k'm_3m_4$, we have $f_{24} : f_{34} = m_2 : m_3 = n$, as was found by actual measurement.

63. **Unit strength of pole.**—In order to be able to deduce the strength of a pole as directly as possible from magnetic forces which it exerts, the unit strength of pole must be expressed in terms of an absolute unit of force, the dyne (§ 57). It will be convenient, therefore, to adopt the following definition: *a magnetic pole has the unit strength when it repels an equal pole at a distance of one centimetre with a force of one dyne.*

Thus two unit poles one centimetre apart exert on one another a force 1/981 as great as the earth's attraction on one gram of matter at its surface.

If a given pole exerts upon a unit pole at a distance of 1 cm. a force of m_1 dynes, we say that the strength of the pole in question is m_1. If it acts upon a pole whose strength is m_2, the force will be equal to $m_1 m_2$ dynes. Now let the distance between the poles be changed from 1 cm. to r cm. We shall then have for the force exerted by either pole on the other

$$f_{12}=\frac{m_1 m_2}{r^2} \text{ dynes} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

In the old theory, the factors m_1 and m_2 denoted the 'quantities of free magnetic fluid' at the two poles. The greater the effect which the poles produce, the greater is their strength, and the greater is the number by which the strength is measured. When there is no magnetism, the strength of pole is zero.

Equation (2) is the complete symbolical expression of the so-called *Law of* COULOMB.

64. **Signs of the poles; the unit pole.**—The strengths of the poles of a uniformly magnetised bar are equal in magnitude but opposite in sense; the one emits exactly as many lines of force as the other absorbs.

To indicate the polarity a positive or negative sign is placed before the numerical measure of the strength. Positive and negative numbers are opposite to one another, just as are north poles which emit lines of force, and south poles which absorb them. It is usual to attribute positive values to north poles, and negative values to south poles.

Now on multiplying together two quantities which are both positive or both negative we obtain a positive product, while we know that two poles of the same name repel one another. And again, two quantities of which one is positive and the other negative give on multiplication a negative product, while poles of unlike name attract one another. Thus our formula (2) gives the sense as well as the magnitude of the force, provided we reckon a force of repulsion positive and a force of attraction negative.

In magnetic investigations an important part is played by the so-called *unit pole*, that is the north pole of a thin bar-magnet, whose length is so great that the field due to one pole is not sensibly disturbed by the presence of the other, and which is magnetised to such an extent that the strength of its pole is exactly equal to one absolute unit.

Such a pole may be approximately realised in practice by suitably magnetising a long thin knitting-needle, by stroking it with a magnet. The strength of pole will then be $m=1$ absolute unit.

65. **Dimensions of strength of pole.**—Since we know the dimensions of force in terms of the fundamental units, we may deduce those of strength of pole from the formula (2), which expresses a relation amongst quantities of the nature: force, length, strength-of-pole. In the special case where the strengths of the two poles concerned are equal and equal to m, the relation (2) becomes

$$r^2 f = m \cdot m \text{ or } m = r f^{\frac{1}{2}}.$$

The force f has the dimensions $[L][M][T]^{-2}$, so that its square root has the dimensions $[L]^{\frac{1}{2}}[M]^{\frac{1}{2}}[T]^{-1}$, while r has of course the dimension [L]. Hence the dimensions of strength-of-pole are

$$[L]^{\frac{3}{2}}[M]^{\frac{1}{2}}[T]^{-1}$$

These fractional exponents show that if we are to speak of 'magnetic masses' we shall encounter some difficulties of conception. Such masses would not be related in any simple way to masses of ordinary matter, which can be measured in grams.

66. **Determination of strength of pole in absolute measure.** We may conveniently proceed as in § 58, measuring in dynes the force which one pole of the bar to be examined exerts upon one pole of a similar bar magnetised to the same extent; the force in question being determined by means of the balance, by finding how many grams have a weight equal to it in amount. If we measure the distance between the mutually acting poles, we can calculate from the law of inverse square what the attraction would be at a distance of 1 cm. This attraction is numerically equal to the product of the strengths of the two equal poles, and thus its square root gives the value required.

In experiment 35, let the thickness d of the glass plate be determined; this is the quantity r of our formula, the distance between the mutually acting poles. From f and r we obtain the value of m.

Numerical example.—Thickness of glass plate $d=0{\cdot}184$ cm., and accordingly strength of pole $m=d\sqrt{f_{12}}$=about 2 absolute units; *i.e.* $m=2$ cm.$^{\frac{3}{2}}$ gr.$^{\frac{1}{2}}$ sec.$^{-1}$

67. **Magnetic moment.**—When the places on a magnet from which lines of force proceed or to which they return are approximately points, the forces exerted on the magnet by external magnetic fields will have these points for their points of application. The rotatory influence of the field upon the magnet is determined by the distance between the two poles, as will be shown in § 74 below.

The strength of either pole m, multiplied by the distance a between the two poles, is called the magnetic moment M of the bar magnet; or

$$M=ma \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3).$$

If the magnet is very long and thin (its length being about forty times its diameter) we may assume that its poles are at its two ends.

In this case $a=l$, the length of the bar.

For bar magnets which are shorter and thicker, the distance between the poles is about $\frac{5}{6}$ of the length of the bar: $a=\frac{5}{6}l$.

The knitting-needles, the strength of whose poles were determined in § 66 as about 2 absolute units, had a length of 17 cm. Their magnetic moments are thus equal to 34 units: M=34 absolute units of magnetic moment.

The dimensions of magnetic moment are:

$$[L]^{\frac{3}{2}}[M]^{\frac{1}{2}}[T]^{-1} \times [L]=[L]^{\frac{5}{2}}[M]^{\frac{1}{2}}T^{-1}$$

The moment per unit mass of a bar is called its specific magnetism; in ordinary steel bars it seldom exceeds 40 absolute units. In the above example, where the mass of the needle is 9·23 grams, there are only three or four units of specific magnetism.

68. Conception and measurement of strength of field.—If we make exception of the few special cases where we are dealing with long and thin bar magnets, the action of a magnetic field cannot be referred to poles situated at determinate points; there are always more extended regions from which lines of force proceed. It will be best, then, to deal directly with the distribution of lines of force in the field, without attempting to refer the magnetic influence to definite centres. So far we have only inferred from the lines of force the relative magnitudes of the magnetic forces at different places. If we suppose, however, that a unit pole +1 is brought into the field at a point where the strength is to be determined, we shall be able to obtain the strength of field in absolute measure. The pole indeed produces a slight disturbance of the lines of force throughout the field;[1] but we assume that the lines of force are very numerous, so that the few lines introduced by the comparatively weak pole +1 produce but little disturbance except quite near to the pole itself. The corresponding

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[[1] This, however, would in no way invalidate the measurement.]

negative pole −1 (of the same bar magnet) is supposed to be far enough off to be without appreciable influence. The pole +1 experiences a force due to the tension along the lines of force and the pressure across them, and this force, measured in dynes, gives the direction and magnitude of the magnetic force at the point in question. At a given point in a given field the force exerted on a pole of strength m will be m times as great as that on the pole +1. The magnetic force within any field may thus be completely determined if we adopt the following definition of the absolute unit of measurement:

The strength of field, or intensity of field, at any point is measured by the force in dynes exerted upon a unit pole placed at that point.

Following MAXWELL, we shall denote this quantity by $\mathfrak{H}$.

Experiment 37.—Suspend from the shorter scale of a hydrostatic balance one of the bars mentioned in § 64, and add weights to restore equilibrium. Approximate to this bar from below the pole of a vertically supported bar magnet, and let the weights which must now be added to restore equilibrium be $p_1, p_2 \ldots$ grams, corresponding to the distances $r_1, r_2, \ldots$ between the attracting poles. The field intensities at points along the axis of the bar magnet and at distances r_1, r_2, from the pole are thus $981\,p_1$, $981\,p_2, \ldots$ units.

We shall find later that the intensity of the earth's magnetic field in this part of the world is about 0·5 such units. Thus the north-seeking pole +1 will be acted upon by a force of about half a dyne directed obliquely downwards (parallel to the axis of the dipping-needle), the south-seeking pole −1 being acted upon by an equal force directed obliquely upwards. Since a dyne is approximately equal to the weight of a milligram, the forces here specified will be equal to the weight of about half a milligram, and therefore very small compared with the weight of the needle itself. We may say then, generally, that the earth's magnetic field gives rise to much smaller forces than does its gravitational field.

69. Mechanical force exerted upon a pole of determinate strength by a field of given intensity.—Our definition of field-intensity leads at once to the value of the force exerted on a magnetic pole of *any* strength m. We shall suppose that, as in the case of the pole $+ 1$, the field is sensibly unaltered by the introduction of the pole m.[1] If the former pole experiences a force of $\mathfrak{H}$ dynes (the measure of the given field-intensity), the force acting on the pole m will be $f = m\mathfrak{H}$ dynes, in the direction of the lines of force.

70. Dimensions of field intensity.—We estimate the intensity of a field by the effect produced upon a pole of determinate strength introduced into the field. From $f = \mathfrak{H}\, m$ it follows that $\mathfrak{H} = f/m$. The dimensions of field-intensity are to be found by dividing those of force by those of strength of pole. Thus the dimensions of $\mathfrak{H}$ are:

$$[L]\,[M]\,[T]^{-2} \div [L]^{\frac{3}{2}}\,[M]^{\frac{1}{2}}\,[T]^{-1} = [L]^{-\frac{1}{2}}\,[M]^{\frac{1}{2}}\,[T]^{-1}$$

Corresponding to the relation $f=\mathfrak{H}m$ for a magnetic field, we have in the case of gravity the weight $p=g\mu$, where g is the acceleration of a freely falling body, and μ is the mass of the body whose weight is p. Thus the acceleration g measures the intensity of the earth's gravitational field.

The difference in the dimensions of g and $\mathfrak{H}$ is due to the difference between the quantities with which the force f is associated; the gram in the one case, the unit pole in the other.

71. Determination of field-intensity from the number of lines of force.—The introduction of a unit pole into the field and the measurement of the force exerted upon it is a fiction like the unit pole itself, and is only intended to make clear the conception of field-intensity. The practical application of the principle, quite apart from other difficulties, would always be upset by the circumstance already mentioned; the introduction of the unit pole deforms the field, so that we never measure it as it really is.[2]

For the comparative estimation of field-intensities throughout any given region, we have already made use of the number of lines of force which cut through an area of 1 cm.^{2} taken perpendicularly to their direction. It was Faraday who developed this method of counting the lines of force into an exact method for determining the intensity of a magnetic field at any point, in accordance with the following rule:

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[1] *Cf.* footnote, p. 88.

[2] But see footnote, p. 88.

Of the indefinitely numerous lines of force which we may consider actually to exist, we shall suppose that at each place there are only as many per square centimetre as there are absolute units in the strength of the field; or, in other words, *where each square centimetre is intersected perpendicularly by one line of force, we say that the field has unit strength, while if* $\mathfrak{H}$ *lines of force cut perpendicularly through the unit of area the strength of field is* $\mathfrak{H}$ *cm.*$^{-\frac{1}{2}}$ *gr.*$^{\frac{1}{2}}$ *sec.*$^{-1}$.

That this rule may be applicable, we must have some definite way of mapping out the lines of force. We shall describe in the next paragraph an ideal construction for the simplest kind of magnetic field, namely that of a point-pole, reserving till later the discussion of more complicated cases.

To represent the earth's gravitational field in these latitudes in accordance with the above convention, we must draw through a horizontal area of 1 sq. cm., 981 vertical lines.

72. **Number of lines of force proceeding from a pole of given strength.**—From a pole of a thin bar magnet there are in reality an infinite number of lines of force, distributed impartially in all directions, and each following a rectilinear course. But, in accordance with the rule given in the last paragraph, we shall find it best to represent the field by means of a limited number of lines. We shall calculate how many lines of force must be supposed to issue from a pole whose strength is m absolute units.

In accordance with the law of COULOMB, this pole exerts upon a pole of strength $+1$, placed at the distance r, the force m/r^2. That is, at the distance r the field due to the pole m has the strength

$$\mathfrak{H} = \frac{m}{r^2}$$

This value $\mathfrak{H}$ of the field-intensity obtains over the surface of a sphere of radius r, that is, over an area $4\pi r^2$. Now at every place where the strength of field is $\mathfrak{H}$, we have to think of $\mathfrak{H}$ lines of force cutting through each square centimetre of (perpendicular) surface. The spherical surface with which we are now concerned is thus intersected by $\mathfrak{H} \times 4\pi r^2$ or $4\pi m$ lines of force. Since we have supposed that there are no magnetic poles present, besides that which we specially consider, no lines of force can originate or disappear on the way from the pole to the spherical surface ; so that we must say :

From a pole whose strength is m *cm.*$^{\frac{3}{2}}$ *gr.*$^{\frac{1}{2}}$ *sec.*$^{-1}$, *the number of lines emerging is* $4\pi m$.

Since $\pi = 3{\cdot}14159 \ldots$, it follows that roughly about $12\frac{1}{2}$ times as many lines of force diverge from a pole as there are absolute units in the strength of the pole. If the pole is negative the lines will converge to it, and there terminate. And conversely if we have plotted the lines of force throughout the field in accordance with this rule, we can calculate at once the strength of the pole from the number of lines which pass, in one direction or the other, through any given area of the diagram ; that is, from the capacity of the source or sink, as Maxwell terms it.

73. Deflection of a declination needle by a bar magnet.—If we bring near to a declination needle a bar magnet of moment M, in the first or second of the positions referred to in § 38, a deflection of the needle will be produced. The deflecting couple we may consider to arise from the action of the two pairs of magnetic poles upon one another, the resisting couple being due to the horizontal component H of the earth's magnetism. The mutual action of the poles may be calculated from COULOMB's law. The needle, being deflected, rests in such a position that all the forces acting upon it are in equilibrium. If the lengths of the needle and the bar magnet are small compared with the distance r between their centres, the deflection is approximately proportional to the inverse cube of the distance r. Supposing the needle to be deflected through an angle ϕ, and equating to zero the resultant of all the moments acting upon it, we obtain

$$\text{In the first position: } M/H = \tfrac{1}{2}\, r^3 \tan\phi$$
$$\text{In the second position: } M/H = r^3 \tan\phi$$

These relations may be verified by means of a device of WILHELM WEBER. A straight groove is cut in a length of wood, and is graduated. At the middle of the length of the groove is placed a compass needle, and on setting the apparatus so that the groove is either perpendicular to, or lying in, the magnetic meridian plane, the deflections corresponding to the first and second positions may be observed. The moment of the declination needle disappears from the calculation.

B.—Dynamical methods of measuring magnetic magnitudes; method of oscillations

Magnetic magnitudes may be measured not only by observing deflections when equilibrium has obtained, but by studying the oscillations which result when a body maintained in equilibrium by magnetic forces has been slightly disturbed.

74. **Couples acting upon a magnetic needle in a uniform field.**—In a uniform field of strength $\mathfrak{H}$, a magnetic needle free to turn about an axis perpendicular to the lines of force always sets itself in the direction of those lines. If m is numerically the strength of pole, then on each pole we have the force $\mathfrak{H} m$ dynes. If the needle is turned into a position at right angles to the direction of the lines of force, and if a is the distance between its poles, we have the force $\mathfrak{H} m$ acting at one pole on an arm of length $a/2$ and giving rise to a restoring moment $\mathfrak{H} m a/2$, while an equal moment tending in the same direction of rotation arises from the force acting on the other pole. Thus altogether the moment tending to bring back the needle to its position of equilibrium is $2\mathfrak{H} m\, a/2 = \mathfrak{H}\, ma$. Here $ma = \mathrm{M}$, the magnetic moment of the needle (§ 67). If the needle makes an angle ϕ with the direction of the lines of force, there is only a component of the force $\mathfrak{H} m$ on each pole which contributes to the restoring moment, that component, namely, which is perpendicular to the length of the needle, while the equal and opposite components of force along the needle itself are in equilibrium with one another. At each end of the needle, then, there is a perpendicular component of force $\mathfrak{H} m \sin\phi$; so that the moment tending to restore the needle to its position of rest is

$$\mathfrak{H}\,\mathrm{M}\sin\phi$$

This expression is similar to that which gives the restoring moment in the case of an ordinary pendulum which has been disturbed from its vertical position of rest. Here in place of $\mathfrak{H}$ we have the intensity of the earth's gravitational field at the place of observation, that is, the acceleration g; in place of the magnetic moment M we have the mass of the pendulum multiplied by the distance of its centre of gravity from the axis of suspension. A magnetic needle, when disturbed from its position of rest in a uniform field, must therefore execute oscillations exactly resembling those of a pendulum.

75. **Comparison of field-intensities by the method of oscillations.**—One of the laws of the pendulum tells us that the square of the number of oscillations per unit of time is proportional to the intensity of the forces which act on the pendulum. If the magnetic moment M of the vibrating needle is not subject to variation, we may use the method of oscillations to compare the intensities at different parts of the same field.

By the number of vibrations we are to understand the number of whole vibrations, that is of transits through the position of equilibrium *in a determinate sense.* As an example, we will apply the method of oscillations to prove COULOMB's law of the inverse square of the distance.

Experiment 38.—A strongly magnetised steel bar, over a metre in length, is fixed in a vertical position, the direction of the magnetic meridian being marked out on the ground. In a horizontal plane, about $\frac{1}{10}$ or $\frac{1}{12}$ of the length of the bar below its upper end, a short magnetic needle suspended from a cocoon fibre is allowed to oscillate; the position chosen for the needle being magnetic north or south of the bar magnet according as this latter has its north-seeking or its south-seeking pole uppermost. The square of the number of oscillations executed by the needle in unit time is determined, as well as the distance r from the axis of the bar to the centre of the needle. Finally the bar magnet is removed, and the needle allowed to oscillate freely in the earth's magnetic field alone, the square of the number of oscillations per second in this case being a measure of the horizontal component of the field, while in the other observations this horizontal component was superadded to the field of the bar magnet. Accordingly we are to subtract the squared vibration frequency obtained in the last case from the corresponding values previously observed, and the remainders will be proportional to the values of magnetic force due to the bar, that is inversely proportional to the square of the distance *r*.

76. **Quantities which determine the period of oscillation of a needle.**—The nature of an ordinary pendulum, so far as its oscillations are concerned, is completely determined when we know two quantities. One of these is the product M obtained on multiplying together the mass of the pendulum and the distance of its centre of gravity below the axis of suspension. The other (I) is the so-called moment of inertia, and depends on the nature and distribution of the matter composing the pendulum. It is equal to the sum of all the products obtained when every element of mass is multiplied by the square of its distance from the axis of suspension.

The expression for I involves measurements of mass, as well as those geometrical measurements which serve to determine the distances of the component particles from the axis of rotation. The moment of inertia of a rectangular prismatic bar of length *l* and breadth *b*, about an axis through its centre at right angles to the plane of *l* and *b*, is

$$\mathrm{I} = \frac{l^2+b^2}{12}\,\mathrm{P.\ cm.}^2\,\mathrm{gr.};$$

where P is the mass in grams, which we suppose homogeneously distributed. For a cylindrical bar of length *l* and radius ρ, we have

$$\mathrm{I} = \left(\frac{l^2}{12}+\frac{\rho^2}{4}\right)\mathrm{P.\ cm.}^2\,\mathrm{gr.}$$

with respect to an axis through the centre of the bar perpendicular to its length.

The period of oscillation of a suspended magnet also depends upon its moment of inertia; we shall show experimentally that the ratio of the quantity $\mathfrak{H}$M to the moment of inertia I determines the period in question.

Experiment 39.—Magnetise a considerable number of equal and similar needles, and select three that are equally magnetised, testing the equality in the following way. A short piece of straw is fastened with wax to one end of a long cocoon fibre, from which it hangs suspended. The needles are taken in turn, and each one is stuck to the straw, so as to be free to oscillate in a horizontal plane, the number of oscillations in some definite time being observed. If there are three of the needles whose magnetic moments (M) are all equal, the same number will be obtained in the three cases. Now take two of these needles at the same time and afterwards all three, fastening them side by side to the straw, with their north-seeking poles in the same sense, and then allowing them to oscillate. The moment of inertia of the straw and wax may be neglected in comparison with that of the needles. When a single needle is thus replaced by two or three, the moment of inertia of the mass to be set in motion is doubled or trebled, but the magnetic moment of the system is likewise increased in the same proportion, there being two or three poles of strength m to be acted upon by the earth's field whose horizontal component is H. The moment of the forces acting on the vibrating system has thus been doubled or trebled.

Our experiment shows that the frequency of vibration is unaltered; so that it can only depend on the ratio of MH to I.

A more rigorous investigation shows that the frequency of vibration n is given by the expression

$$n^2 = \frac{1}{4\pi^2} \frac{\mathrm{M\,H}}{\mathrm{I}} \ (\text{sec.}^{-2}) \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

77. **Determination of field intensities in absolute measure by oscillations.**—The strength m of the poles of a thin bar magnet may be determined by means of the balance, and the magnetic moment $\mathrm{M} = ma$ calculated. If the bar is cylindrical, of length l cm. and radius ρ cm., the moment of inertia can be calculated from these measurements, and the known mass P (§76). The frequency of oscillation n gives the strength of field $\mathfrak{H}$ in accordance with equation (4).

Experiment 40.—*Determination of the intensity of the earth's magnetic field in absolute measure.*—Let a needle of known magnetic moment M be fastened at its middle to a fine fibre; it will set itself in the direction of the earth's magnetic lines of force. Now disturb the needle from its position of rest and allow it to oscillate. From the frequency of oscillation we can calculate T, the resultant intensity of the earth's magnetic field, or 'total force'; and we shall find in Germany approximately

$$T=\tfrac{1}{2}\ \text{cm.}^{-\frac{1}{2}}\text{gr.}^{\frac{1}{2}}\text{sec.}^{-1};$$

that is, there is half a line of force to each square centimetre of cross-section, or one line of force to each two square centimetres. The model, fig. 19, in which 35 lines of force pass through an area of 70 sq. cm., thus represents the magnetic field quantitatively, provided we give to it the proper orientation.

78. Determination of magnetic moments in absolute measure by oscillations.—We have calculated the total intensity T of the earth's magnetic field. On multiplying this by the cosine of the angle of dip, we obtain (§ 41) the horizontal component of the field, which is approximately

$$H = 0{\cdot}18\ \text{cm.}^{-\frac{1}{2}}\ \text{gr.}^{\frac{1}{2}}\ \text{sec.}^{-1}$$

for central Germany [and also for London]. Knowing this value, we can now find conversely (§ 76) the magnetic moments of given bar magnets, and hence the strengths of their poles. We have only to determine the frequency of oscillation when the magnets are allowed to oscillate in a horizontal plane under the influence of the earth's horizontal magnetic component.

Example. -Each of the rectangular prismatic bar magnets used in producing the line-of-force figures 10, 11, 15, 16, 17, 18, has a length $l=7{\cdot}8$ cm., a breadth $b=0{\cdot}4$ cm., and a mass $P=22{\cdot}05$ grams. When any one of them is suspended by a fine cocoon fibre, it makes six complete oscillations in a minute under the earth's horizontal component $H=0{\cdot}18$. The oscillation frequency is in this case $n=0{\cdot}1$. From the formula (4) we have $M=4\pi^2n^2/IH=246$. Since the distance between the poles $a=\frac{5}{6}l=6{\cdot}5$ cm., and $M=ma$, the strength of either pole of this magnet is $m=38$ units $(\text{cm.}^{\frac{3}{2}}\,\text{gr.}^{\frac{1}{2}}\text{sec.}^{-1})$.

79. Determination of the horizontal component of the earth's magnetic field in absolute measure. (Method of Gauss and Weber.)—The method given in § 77 for determining the intensity of the earth's magnetic field has the disadvantage that the moment of the bar magnet employed is involved in the result. But the magnetisation of steel magnets changes considerably with the temperature, and it is therefore important to devise a method by which the magnetic moment is finally eliminated. Such a method is that of Gauss and Weber. It combines the oscillation method, giving the value A of the product M H, with the deflection method of § 73, giving the value B of the quotient M/H; the magnet used for deflecting the declination-needle being that which has been made to oscillate freely in the earth's magnetic field. On division we obtain

$$H = \sqrt{\frac{A}{B}};$$

the magnetic moment M disappearing from the result.

The expressions A and B only involve such quantities as may be expressed directly in terms of the fundamental units of length, mass, and time. Thus we have the value of the important magnitude H expressed in absolute measure.

CHAPTER V

THE REPRESENTATION OF FIELDS OF FORCE

In the last chapter it was shown how magnetic fields could be measured in absolute units. We have next to consider how they may be represented geometrically, the method which we shall follow being founded on a series of important conceptions which were introduced by **Faraday** and **Maxwell**, and to which we must now turn our attention. The methods for mapping out a field in a plane are of great importance, especially in technical applications, and with these we shall deal first, confining our attention to a few simple but typical problems.

A.—Conceptions of Faraday and Maxwell

In all modern representations of magnetic and electrical phenomena the conceptions of bundles of lines of force, tubes of force and flux of force play a prominent part. But when we here introduce these conceptions, it must be remembered that for the present their significance is purely geometrical. They merely provide us with a construction for mapping out a field in a more convenient way. But they have also a physical significance, for they are an expression of that doctrine of continuity of the magnetic condition which is the foundation of all modern theories, so that they are of the greatest assistance in making clear our physical conceptions.

80. **The bundle of lines of force.**—So far we have only spoken of the separate lines of force; but we must now consider collectively a group of neighbouring lines of force

running all in the same direction. If the direction of the lines of force were everywhere strictly the same, a bundle of such lines would not spread out sideways as it passed through the field. But this disposition of lines of force exemplifies most simply the transmission through the medium of the tensions and pressures which constitute the magnetic field, though the uniformity which we assume is only possible through a limited region of space. If we consider collectively a number of lines of force, we have then a kind of rope or bundle of lines, of finite cross-section. This consists of lines of force which travel through neighbouring parts of space, so that if the cross-section of the bundle is small, we may assume that all the lines composing it lie appreciably in the same direction. Any given bundle of lines of force is, throughout its entire length, made up of the same set of lines. It is only where magnet-poles exist that lines of force can leave a bundle, or new lines enter it.

We have no direct evidence to show that the magnetic lines of force have any definite cross-section, even of very small extent. But in the case of electrical lines of force, which are in many respects analogous, we find that at each atom, or rather at each unit of chemical valency, a certain number of the lines of force terminate, no more and no less. This fact is of the very greatest importance, and well worthy of attention from the point of view of the analogy between electrostatic and magnetic phenomena.

81. **Tubes of force.**—From the bundle of lines of force we pass on to the conception of a 'tube of force,' which is of the highest importance throughout the whole theory of the subject. Imagine a closed curve drawn anywhere within the magnetic field, and not itself in the direction of the lines of force (fig. 24). In the figure the lines of force l_1, l_2, l_3 which pass through the various points p_1, p_2, p_3 of this curve are prominently shown. These lines taken collectively constitute an infinitely thin-walled tube, which encloses a bundle of lines of force. On the outer wall of the tube lie the points p_1, p_2, p_3, and consequently the closed curve from which we started. The generating lines

of the bounding surface are our lines of force l_1, l_2, l_3. . . . *A narrow region thus bounded by lines of force is called a tube of force.*

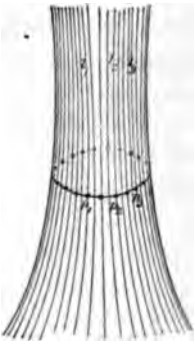

Fig. 24

Maxwell calls it a sphondyloid (from the Greek σφόνδυλος, a vertebra, so that sphondyloid means essentially a narrow space like that which is filled by the spinal chord). The spinal chord was taken to represent the seat of the force, and hence the choice of the term in question. The name 'solenoid,' which is used in the same general sense by **Maxwell** and other English writers, we shall only employ with a very special meaning.

We can conceive of the entire field as divided up into such tubes of force, which originate at north poles and terminate at south poles, as is the case with the lines of force themselves. Where the lines of force spread out from one another, so that the field-intensity is small, the cross-section of the tubes is greatest; where the lines are crowded closely together, so that the intensity is great, the cross-section of the tubes is least. For the bounding surface of each tube, as well as the space within, consists always of the same lines of force.

82. **Flux of force.**—Since in those parts of the field which contain no magnet-poles no lines of force originate or terminate, it follows that along each tube of force the number of lines cutting through the cross-section is constant. The number of lines in question is thus a quantity characteristic of the tube, and it may be easily calculated. If $\mathfrak{H}$ is the field-intensity at some place through which a given tube of force passes, then $\mathfrak{H}$ is also equal to the number of lines of force which at this place cut perpendicularly across the unit of area. If the cross-section of the tube is ω cm.^{2} (taken perpendicularly to the direction of the lines of force) there will be $\mathfrak{H}\,\omega$ lines of force cutting through it.

In describing the properties of the lines of force, we have already used terms which are borrowed from the subject of fluid motion, such nomenclature being especially suitable because the lines of force, as we follow them through the field, exhibit just that kind of continuity which characterises the flow of an incompressible fluid. The same analogy is used in the following definition:

The product of the field-intensity $\mathfrak{H}$ and the cross-section ω of the tube of force is called the flux of force through the tube at the place where $\mathfrak{H}$ and ω are measured:

$$F = \mathfrak{H}\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The magnetic force is not here regarded as something which flows along, but its propagation from place to place through the medium is called the flux of force.

Since $\mathfrak{H}$ is measured by the number of lines of force which cut normally through a square centimetre of surface, $F = \mathfrak{H}\omega$ is the number of lines contained in the tube.

83. Law of the conservation of the flux of force.—Let the flux of force through a tube at a given place be $F = \mathfrak{H}\omega$. If the cross-section at some other place is ω' (greater than ω, let us say), and the strength-of-field $\mathfrak{H}'$, the corresponding flux of force will be $F' = \mathfrak{H}'\omega'$.

If there are no 'sources' or 'sinks' between the two places, that is to say, no magnet-poles, there will be no gain or loss in the number of lines of force as we pass from one place to the other. The same number of lines which previously intersected the area ω is now spread over the area ω', so that the number of lines per unit of cross-sectional area is changed in the ratio $\omega : \omega'$, the field-intensity $\mathfrak{H}'$ having become less in proportion as ω' has become greater. Hence

$$\mathfrak{H}'\omega' = \mathfrak{H}\omega, \text{ or } F' = F.$$

The flux of force along a tube does not change between two cross-sections which include no magnet poles between them.

The flux of force is conserved. The value $F = \mathfrak{H}\omega$ obtained at any one place holds good for the whole tube, F being a magnitude characteristic of the tube in question.

In the hydrodynamical analogue F is the quantity of fluid flowing through a given cross-section per unit time. If the fluid is incompressible, the same quantity passes through each cross-section of a 'tube of flow'; $\mathfrak{H}$ corresponding to the velocity of the fluid.

84. Maxwell's unit solenoids.—Tubes through which the flux of force is unity are called unit tubes. Maxwell introduced the name solenoid (*σωλήν*, a tube) in this connection, but we shall reserve the name for the unit tubes just defined, calling them 'unit solenoids,' or simply solenoids. When the field is mapped out into unit tubes, we obtain a very convenient general view of the distribution of field-intensity. The cross-section of the solenoids need not be circular, as in fig. 24; any strip of space which is bounded laterally by lines of force, and through which the flux of force is unity, is equally a solenoid.

Since the flux of force through a solenoid is unity, its cross-section in cm.^{2} at any place is the reciprocal of the corresponding field-intensity. Where the intensity is less, the tube is wider, while in places of greater intensity the tube becomes contracted. Again, since a positive pole of strength m emits, according to our conventions, $4\pi m$ lines of force, its entire flux of force is also measured by $4\pi m$, and consequently $4\pi m$ solenoids originate at the pole. Thus the solenoids inform us directly of the values of the various sources and sinks in the field.

B.—Flux-of-force diagrams

We will now carry out for some special cases the division of the field into unit solenoids explained in the last paragraph. If the walls of the tubes are of such a form that the cross-sections can be easily calculated, we have at once a convenient means of estimating the field-intensity at each place. But even then a difficulty arises when we attempt to represent a three-dimensional figure by means of a plane diagram. The simplest case is that in which a diagram representing a sectional view is supposed to be rotated about a suitable axis, and so to develop a representation of the entire structure of the field.

85. Diagram of a unipolar field.—Let a pole whose strength in absolute measure is $m = 24/4\pi = 1{\cdot}910$ unit be surrounded by a homogeneous medium. In accordance with §72, the number of lines of force emitted is $4\pi m = 24$, or the total flux of force proceeding from the pole is equal to 24 units. The space surrounding the magnet-pole must therefore be partitioned into 24 (conical) spaces, through each of which the flux of force is unity. The method of effecting this partition is shown in fig. 25, table I (at the end of the book). Through the pole whose strength is 1·910 unit, a straight line is drawn in any direction to serve as axis, and a circle of any radius is described about m as centre. The axial diameter is now to be divided into as many equal parts as there are units in the flux of force from the pole : in our case 24. Through each of the 23 points of division, a straight line is drawn perpendicular to the axis, and produced both ways until it cuts the circle, the points of intersection being joined by straight lines to the centre. If the whole figure were to be rotated about the axis, the circle would describe a sphere, while the radii which we have drawn would describe conical surfaces, having their common vertex at m, and dividing space into equal compartments. The zones which these cones cut out from the surface of the sphere are all equal in area, in accordance with well-known geometrical principles.

Since the flux of force 24, which emanates from the pole m, spreads indifferently in all directions, it follows that each unit of surface of the sphere is traversed by the same flux of force, so that the flux through each of our 24 compartments will be exactly unity.

Thus through each of the 24 conical regions into which the surrounding space is divided by the rotation of the figure, the number of lines of force passing is the same.

In the figure, then, the lines of force represented are just those which form the boundaries between bundles of equal numbers of lines.

86. Estimation of field intensity from the diagram.—If the diagram is drawn exactly to the theoretical scale, with the centimetre for the unit of length, or if we know what the scale of the diagram is (in fig. 25 it is appended), we have immediately the means of determining the field-intensities throughout the region, with the aid of a pair of compasses and a little calculation.

If at any place we measure the breadth b between two adjacent lines of force in the diagram, and the distance r of this place from the axis, the annular strip described by b when the figure rotates about its axis will have an area equal to $2\pi rb$.

The line b which measures the breadth between the two lines of force (compare the dotted lines at the places a and b in fig. 25) meets them very nearly at right angles.

Through this area $2\pi rb$, the flux of force is unity; hence,

$$\mathfrak{H} = \frac{1}{2\pi rb} \text{ absolute units.}$$

At the place a near to the pole m ($=1\cdot910$ unit) we find $r=3\cdot4$, $b=0\cdot8$ unit of length, by setting off these lengths against the scale shown in the diagram, by means of a pair of compasses. This gives for $\mathfrak{H}$ the approximate value $0\cdot059$ cm.$^{-\frac{1}{2}}$gr.$^{\frac{1}{2}}$sec.$^{-1}$. The direct distance from the pole m is $5\cdot7$ cm.; so that the law of inverse square would give $\mathfrak{H}=1\cdot910/(5\cdot7)^2=0\cdot059$, as before. At the place b, more distant from the pole, we have $r=16\cdot2$, $b=1\cdot9$ cm.; so that $\mathfrak{H}=0\cdot0052$ cm.$^{-\frac{1}{2}}$gr.$^{\frac{1}{2}}$sec.$^{-1}$, while the direct calculation gives $\mathfrak{H}=1\cdot910/(19\cdot3)^2=0\cdot0051$.

Our method of evaluating field-intensities from the diagram does not appear in the present case to offer any special advantage; but that is because the relations are so simple. We could equally well measure the distance of the place in question from the pole, and divide the known strength of pole by the square of this distance. The value of the method will, however, become more apparent when we come to deal with more complicated cases, where the field is due to a superposition of the effects due to two or more poles, as in the next example.

At each place in the diagram of a single pole, the strength of field is inversely proportional to the area of a certain annular strip whose breadth we have called b. But the adjacent lines of force whose distance apart is measured by b diverge from one another as we follow them further from the pole, and the divergence is not the same for all. Instead of merely marking out the generating lines of the cones which partition the surrounding space into unit tubes, we may make a further sub-division, so that the field is divided up into more than $4\pi m$ compartments.

If in fig. 25, for example, we wished to carry the subdivision to such a point that through each conical compartment the flux of force was $\frac{1}{10}$ of a unit, we should only have to divide the axial diameter into ten times as many (that is, into 240) equal parts, and then to draw the corresponding perpendiculars and radii.

The more numerous we make the lines proceeding from the pole, the less rapidly do any two adjacent lines diverge from one another, and the more nearly may we consider them as parallel through any small part of their length.

As before, the field intensity at each point is inversely proportional to the area of an annular strip whose breadth is the distance apart of two adjacent radii of the diagram.

If b' is this distance at a place whose distance from the axis is r', the area of the annular strip will be $2\pi r'b'$. The flux of force through the strip is $\frac{1}{10}$ of a unit; so that

$$\mathfrak{H} . 2\pi r'b' = \tfrac{1}{10},$$

or

$$\mathfrak{H} = \frac{1}{10 \cdot 2\pi r'b'}$$

87. **Diagram of a bipolar magnetic field.**—Let there be two equal and opposite magnet poles, surrounded by a homogeneous medium. The field in this case was made apparent by means of iron filings in fig. 12, and we shall now reconstruct it by means of a diagram. Let the north pole have the strength $m = +\frac{20}{4\pi} = +1.592$ cm.$^{\frac{3}{2}}$ gr.$^{\frac{1}{2}}$ sec.$^{-1}$, the south pole the strength $-\frac{20}{4\pi} = -1{\cdot}592$

$\text{cm.}^{\frac{3}{2}}\text{gr.}^{\frac{1}{2}}\text{sec.}^{-1}$. We must first construct the diagram for each pole considered separately (see the lower part of fig. 26, plate I). The diameters of the circles drawn round the two poles as centres are divided each into 20 equal parts, since the flux of force corresponding to each pole is $4\pi \times 1{\cdot}592 = 20$. The line joining the two poles is taken as the axis of the diagram, and the line-of-force diagram for each pole is formed by drawing perpendiculars and radii exactly as described in the last paragraph. If we imagine the whole figure rotated about the axis NS, the radii will map out 2×20 conical compartments intersecting one another; through 20 of these compartments there is a unit flux of force from the north-seeking pole into the surrounding medium, through the other 20 there is a unit flux of force from the medium towards the south-seeking pole. The regions thus mapped out must become, as it were, fused together when we consider the simultaneous operation of the source at N and the sink at S. The combination of the separate fields is expressed in the diagram by drawing the diagonals of the small quadrilateral figures which result from the intersection of the two systems of lines of force. This will be further explained in the next paragraph. The diagonals thus drawn form broken lines, connecting the poles together, and may be easily rounded off so as to become continuous curves. We approximate more and more closely to the true form of these curves when we increase the number of sub-divisions in the diagram tenfold, a hundredfold, and so on, the flux of force through each separate compartment being then one-tenth or one-hundredth, or some still smaller fraction of an absolute unit.

On rotating the figure about its axis the 20 curves in question divide up the field into shell-like spaces, each extending from pole to pole, and conveying unit flux of force from the north-seeking to the south-seeking pole. These shell-like spaces are therefore unit solenoids.

The innermost solenoid has not the form of a shell, but rather that of a spindle with pointed ends. The outermost, 20, starts from a cusp at N, and spreads out to infinity towards the right, bending round and finally reaching S in the form of a narrow cusp. [Thus each of the first 19 solenoids occupies a finite portion of space, and the twentieth occupies the infinite remainder.] In the diagram the boundary lines are lines of force, and their form agrees very closely with that shown by means of iron filings in fig. 12; though in the present case we have a representation of a number of determinate lines.

88. Combination of two flux-of-force diagrams.—The line-of-force diagrams for two poles in the same field may be so finely divided up that throughout any small portion of their length neighbouring lines of force may be regarded as parallel (compare § 86). For example, let the two parallel straight lines K_1 (fig. 27) be lines of force proceeding from a north pole, while the other two straight lines K_2 are proceeding towards a south pole. The strength of field $_1$ at the point P due to the north pole is, in accordance with § 86, inversely proportional to the distance b_1, the strength of field $\mathfrak{H}_2$ due to the south pole being inversely proportional to the distance b_2. The pairs of lines K_1 and K_2 enclose a parallelogram whose opposite angles are of course equal; the angle at its highest or lowest corner in the figure being called a. From the similarity of the two right-angled triangles lying between $s_1 b_2$ on the one hand and $s_2 b_1$ on the other hand, it follows that

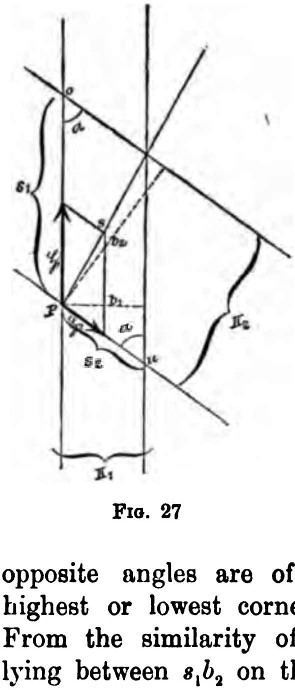

Fig. 27

$$s_1 : s_2 = b_2 : b_1,$$

or

$$s_1 : s_2 = \ _1 : \mathfrak{H}_2.$$

But the parallelogram in the figure formed by the crossing of the two pairs of lines of force has s_1 and s_2 for adjacent sides, so that the resultant of the two field-intensities $\mathfrak{H}_1$ and $\mathfrak{H}_2$ is along that diagonal of the parallelogram which passes through P.

Thus if two line-of-force diagrams are superposed one upon another, the directions of the resultant lines of force are to be found in accordance with the following simple rule: *through each of the elementary parallelograms formed by the crossing of pairs of consecutive lines of foree, draw the appropriate diagonal.*

If this rule were applied to fig. 26, it could only furnish a rough idea of the directions of the lines of force, since here the separate lines of force deviate considerably from parallelism. But if we draw a sufficient number of intermediate lines, corresponding to fractions of the unit flux of force, and then apply the rule to the smaller parallelograms so formed, we shall obtain a result much nearer to the truth.

In fig. 27, if the two poles had been of the same name, we should have had to draw the diagonal joining the highest and lowest points of the parallelogram.

Since a side and a diagonal of a parallelogram are to one another in the inverse ratio of the corresponding heights, we have

$$s_1 : s = \frac{1}{b_1} : \frac{1}{b},$$

where s denotes the length of the diagonal and b its perpendicular distance from one of the angular points o or u. If the resultant magnetic force is $\mathfrak{H}$, it follows from the similarity of the two parallelograms that

$$\mathfrak{H}_1 : \mathfrak{H} = s_1 : s = \frac{1}{b_1} : \frac{1}{b}.$$

Similarly

$$\mathfrak{H}_2 : \mathfrak{H} = s_2 : s = \frac{1}{b_2} : \frac{1}{b};$$

or

$$\mathfrak{H}_1 : \frac{1}{b_1} = \mathfrak{H} : \frac{1}{b} \text{ and } \mathfrak{H}_2 : \frac{1}{b_2} = \mathfrak{H} : \frac{1}{b};$$

whence finally

$$\mathfrak{H}_1 b_1 = \mathfrak{H} b = \mathfrak{H}_2 b_2.$$

When fig. 27 is rotated about an axis through the two magnet poles, the pairs of lines K_1 and K_2 (fig. 27) map out two conical shells, of such cross-sectional area ($2\pi r b_1$) that $\mathfrak{H}_1 b_1 . 2\pi r = 1$ and $\mathfrak{H}_2 b_2 . 2\pi r = 1$; for through each shell the flux of force is unity. During the rotation also, the resultant line s and the two neighbouring lines through o and u map out shell-like spaces for which

$$\mathfrak{H} b . 2\pi r = \mathfrak{H}_1 b_1 . 2\pi r = \mathfrak{H}_2 b_2 . 2\pi r = 1.$$

But the quantity $\mathfrak{H} b . 2\pi r$ represents the flux of force through these last-mentioned shells, from the north pole to the south pole.

89. **Evaluation of the field-intensity at any point of a bipolar magnetic field, by means of the diagram.**—In accordance with the results obtained in the last paragraph, we have only to calculate the cross-sectional area of one of the shell-like solenoids of the bipolar field in order to obtain the resultant field-intensity in absolute measure.

If at a place whose distance from the axis is r, the distance between two adjacent lines of the diagram is equal to b, the cross-sectional area of the corresponding shell is equal to $2\pi r . b$, and the corresponding field-intensity

$$\mathfrak{H} = \frac{1}{2\pi r . b}.$$

At the place marked in fig. 26, which is drawn to the same scale as fig. 25, $b = 3{\cdot}5$ cm., $r = 13{\cdot}2$ cm., so that $\mathfrak{H} = 0{\cdot}0034$ $\text{cm.}^{-\frac{1}{2}}\text{gr.}^{\frac{1}{2}}\text{sec.}^{-1}$. In this case it would have been considerably more trouble to calculate the direction and magnitude of the resultant magnetic force from the law of inverse square.

90. **Diagram for two poles of unequal strength.**—We may plot diagrammatically the field of two magnet poles of unequal strength, just as we have already done for poles of

the same strength. The axis of the diagram is of course drawn through the positions occupied by the two poles. A unipolar diagram is then constructed for each pole separately, and on combining the two we obtain the diagram of the resultant field.

(*a*) *Diagram of two unlike poles.*—A north pole whose strength is $+\frac{20}{4\pi} = 1{\cdot}592$ unit, and a south pole of smaller strength $-\frac{5}{4\pi} = -0{\cdot}3979$ unit are placed at a determinate distance apart. The flux of force from the first is twenty units, while the flux inwards to the second is five units. Fig. 28, plate I, shows the separate diagram for each pole. The resultant lines of force in the combined field are determined as follows:—Consider any point of intersection of two lines of force, one belonging to each system, and imagine a positive unit pole to be placed there; it will be repelled by N and attracted by S, and this sufficiently indicates from which corner of the parallelogram the diagonal is to be drawn. Through the two conical regions which lie closest to the axis, between the two poles, the flux of force passes directly from one pole to the other, since the field-intensities due to N and *s* are there in the same general direction, namely, from N towards *s*. Hence the innermost lines of force are bowed out, curving somewhat away from their original directions, and the same is true of all the remaining lines. In the diagram thus obtained we may observe the following features: The lines of force which proceed from N become bowed out more and more widely, and finally are gathered together at *s*, where they terminate. On rotating the figure, five coaxal shells are mapped out, through each of which the unit flux of force passes over from N to *s*. This constitutes the entire flux of force which *s* is capable of receiving, so that the remaining lines from N do not terminate on *s*, but pass by it. There are thus 15 unit solenoids starting from N, which do not terminate within the diagram, but carry their flux of force to remoter parts of the field.

The entire distribution of the flux of force originating at N is deflected somewhat towards the right by the presence of the unlike pole *s*. Owing to the tension along the lines of force and the pressure across them, the two poles will be urged towards one another (attraction). If the two poles are free to move, they will mutually approach one another.

[The forces which they exert upon one another will be equal and opposite, in accordance with the third law of motion, and it should be noticed that though all the lines of force attached to and pulling *s* are pulling N more directly along the axis, there are also a considerable number of lines pulling N away from *s*.]

Here as before we can calculate at once the field-intensity at any place when we know the breadth b between consecutive lines and the distance r from the axis.

(*b*) *Diagram of two like poles.*—Let the strength of one pole be as before $\frac{20}{4\pi} = 1{\cdot}592$ unit, that of the other pole being $\frac{5}{4\pi} = 0{\cdot}3979$, so that the two poles in this case are of the same name, a north pole N being placed at a determinate distance from a weaker north pole n. At the points of intersection of the lines corresponding to the two unipolar fields, the forces to be compounded are both directed away from their poles. We have a network of intersecting lines precisely like that of fig. 28, but through each of the meshes we have now to draw the other diagonal, so that we obtain the result shown in fig. 29. The lines of force, as they approach one another from the two poles, bend round so as to avoid meeting. There is one line which separates the two systems from one another, and does not pass through either pole. Where this line meets the axis of the diagram, there is an indifferent point, or point of zero magnetic force, J. The 20 units of flux of force proceeding from the pole N all pass outwards into the surrounding space, and so likewise do the 5 units from the pole n, no intermixture of the two taking place.

Owing to the pressure across the direction of the lines of force, there will be a force on each pole urging it away from the other; that is, the two poles will repel one another.

If we suppose pressure and tension to be interchanged, so that there is pressure along the lines of force, and a tension across them, the diagram may be taken to represent a double star, whose members attract one another gravitationally.

91. **Diagrams of homogeneous fields.**—In a uniform field the lines of force are all straight and parallel, and distributed throughout with the same density. If we intend to apply to this case the same principle of construction that was used for systems having an axis of symmetry, we must choose as axis some line parallel to the direction of the field, and divide the space surrounding it into cylindrical shells, each of such cross-sectional area that the flux of force through it is unity. In the plane of the diagram, the limits of these cylindrical shells appear as straight lines parallel to the axis. The distances $r_1, r_2, r_3 \ldots\ldots$ of these lines from the axis must satisfy the simple relation

$$\mathfrak{H}\,\pi r_1^2 = 1,\ \mathfrak{H}(\pi r_2^2 - \pi r_1^2) = 1,\ \ldots$$

or

$$\mathfrak{H}\,\pi r_2^2 = 2,\ \mathfrak{H}\pi r_3^2 = 3\ \ldots\ldots$$

When the diagram has been prepared in accordance with these conditions, it may be combined with the diagram for a single pole by drawing the diagonals of the small quadrilateral figures produced by the crossing of the two systems of lines. With the resultant lines we may then use the formula

$$\mathfrak{H} = \frac{1}{2\pi r b}.$$

In fig. 30 a uniform field is represented, having the absolute strength $\mathfrak{H} = 0{\cdot}00771\ \mathrm{cm}^{-\frac{1}{2}}\,\mathrm{gr}^{\frac{1}{2}}\,\mathrm{sec}^{-1}$. The radii of the successive cylindrical surfaces must accordingly be

$$r_1 = \sqrt{\frac{1}{\pi\mathfrak{H}}} = 6{\cdot}4\ \mathrm{cm.};\quad r_2 = \sqrt{\frac{2}{\pi\mathfrak{H}}} = 9{\cdot}0\ \mathrm{cm.}$$

$$r_3 = \sqrt{\frac{3}{\pi\mathfrak{H}}} = 11{\cdot}1\ \mathrm{cm.};\ \ldots\ldots$$

$$r_9 = \sqrt{\frac{9}{\pi\mathfrak{H}}} = 19{\cdot}2\ \mathrm{cm.}$$

The positive direction of the lines of force is from right to left. We must suppose, therefore, that there are two strong magnet-poles at distances far beyond the limits of the diagram; a north pole to the right and a south pole on the left.

Combined with this field is that of a north-seeking pole of strength $+ m = + \frac{10}{4\pi} = 0{\cdot}7958 \text{ cm.}^{\frac{3}{2}} \text{ gr.}^{\frac{1}{2}} \text{ sec.}^{-1}$. If we suppose a positive unit pole placed at a point where lines of the two systems intersect one another, it will be urged towards the left in the homogeneous field, and at the same time repelled by the pole N. This consideration shows us along which diagonals the resultant lines of force are to be drawn. The 10 units of flux of force which proceed from the pole N are so deflected by the influence of the uniform field, that they are ultimately all passing towards the left. The innermost conical surface has become narrowed to the tubular form, 1, 1, and all the remaining surfaces are correspondingly distorted from their original form. The shells numbered seven to ten, which start from the pole towards the right hand of the diagram, are seen to be entirely bent round, and all the ten shells have so completely acquired the direction of the field that further to the left the uniformity is completely restored.

The originally cylindrical shells of the uniform field spread out as they approach N, and bend round it as if it were an obstacle in their path, or more strictly, as if the uniform field were a uniform flow of fluid, and the pole N a source from which fluid was emanating. The surfaces more remote from N maintain their course with but little deviation. The present example shows very clearly the insight to be obtained from the construction of such diagrams.

If we consider the figure from the point of view of the tensions and pressures accompanying the lines of force, it is clear that N must experience a force urging it through the field towards the left. This is in accordance with the tendency of a north pole to move along the positive direction of the lines of force (compare § 28). The reverse would be the case if we replaced the north pole by a south pole, which behaves as a sink towards the lines of force.

92. **Combination of more than two fields.**—A resultant field mapped out by this method may in turn be combined with another field, and so on; so that by successive applications of our construction we may obtain the diagram for even a very complicated case. The separate diagrams may be conveniently drawn on tracing paper, so that when two are superposed they are both easily visible. They must of course be drawn to the same scale. A very instructive case is that of the bipolar field of a bar magnet, combined with the uniform field of the earth, but this is left as an exercise for the reader.

CHAPTER VI

CONSTITUTION OF MAGNETS AND MAGNETIC FIELDS

So far we have confined our attention to the space surrounding magnetised bodies—that is, to the lines of force themselves. We have seen how the presence of these and their general character may be recognised from certain qualitative properties, and we have shown how to measure the quantities which determine their disposition and relations, and how to construct them graphically from numerical data. We must now attempt to obtain some insight into the constitution of the magnet itself; and we shall find that there are certain peculiarities of structure which must be assumed for all bodies through which magnetic lines of force are passing.

A.—The magnetisation

93. **Magnetism as a molecular property.**—*Experiment* 41. Dip the whole length of a well-magnetised steel knitting-needle in iron filings. Considerable tufts of filings will adhere to the ends, while at the middle, in the indifferent region, there are no filings. Now break the needle at the middle and repeat the experiment with the two halves. In each case filings will adhere to the two ends, even to those which previously constituted the indifferent region. If each half of the needle is broken up into shorter pieces, each of these will be found to be a complete magnet, with two magnetic ends and an intermediate non-magnetic region.

Since the same result is obtained, however far we continue the subdivision, we are led to the conclusion that even the smallest particles of which a magnet is composed are themselves magnets. The smallest portions into which a body can be resolved by physical means are called molecules. Thus we conceive of a magnet as built up of so-called 'molecular-magnets,' magnetic properties being attributed to the molecules themselves. It is an essential feature of the magnetic condition that even the smallest particles of a magnet possess polar properties.

94. **Chains of molecular magnets.**—In accordance with the last experiment, the contrariety of properties of north and south poles, observed in the case of all magnets, must be supposed to extend to the molecules of which a magnet is built up; the north pole of a molecule being a source of lines of force, and the south pole a sink. But the molecules of a magnet cannot have the directions of their magnetic axes distributed in a purely random way. In the neighbourhood of the indifferent zone, for example, the flux of force proceeding from a positive pole must be taken in by a neighbouring negative pole, belonging to an adjacent molecule, or something of this general character must take place; the axes of the little molecular magnets being set more in the direction of the axis of the non-magnetised body than in any other. Towards the ends of the magnet, the general set of the molecular axes is along lines which intersect the bounding surface, molecular north poles being outermost at one end and south poles at the other. Lines of force originate at the former end, and terminate at the latter. The simplest conceivable arrangement of the molecular magnets is one in which they form chains, whose separate members lie all in the same direction, and have unlike poles in contact. Then throughout nearly the whole length of the chain, the flux of force will pass from one molecule to the next, only the first and the last of the series having a free pole, that is, one in contact with the surrounding region, into which it sends forth lines of force, or from which it receives them

Any ordered arrangement of the magnetic elements of a body is called *magnetisation.*

95. **Model of a bar magnet composed of molecular magnets.** The following models will serve to illustrate the structure of a magnet.

1. A number of small equal rectangular blocks are painted red at one end (north-seeking end) and blue at the other (south-seeking end). These are to represent the molecular magnets; they are arranged end to end in chains, and the chains are arranged to fill an approximately rectangular figure, representing the bar magnet. In the middle of the system, the separate constituents of the magnet are to be set with their axes parallel, each one being directly joined to its neighbours before and behind. In the neighbourhood of the ends the directions of the axes are disposed more irregularly, but still so that at one end it is chiefly north poles that point outwards, and at the other, south poles.

2. Into a rectangular board stick several parallel rows of needles with their points directed vertically upwards. From a piece of steel spring cut pieces as many as there are needles, and of length somewhat less than the distance between consecutive needles. At the middle of each piece make an indentation with a centre-punch ; this will cause the two halves to bend round somewhat, so that when the piece of spring is mounted with the indentation supported upon a needle-point, it will not fall off, its centre of gravity being below the point of support. Now magnetise the pieces of spring and mount them upon the needle-points ; they arrange themselves in chains and groups which are more or less stable (EWING). At one end of the system all the poles directed outwards are north-seeking, at the other, south-seeking. Now approach to this model of a bar magnet a declination needle in either of the two principal positions mentioned in § 38, and the same effects will be observed as in the case of an actual bar magnet.

96. **Uniform magnetisation.**—Generally speaking the places of emergence and immergence of lines of force are not confined to the ends of a magnet (compare, for example, fig. 4, p. 17) ; so that even in the interior the chains of

molecules cannot be regularly arranged, and it is only quite near the middle that the general direction of the molecular magnetic axes is along the axis of the bar. This may be regarded as due to a certain 'demagnetising influence of the ends,' which brings about a loosening of the combinations of molecular magnets. In bar magnets, then, especially those with free ends, the magnetisation is more or less irregular.

But we may so order the disposition of the lines of force in the interior, that every volume-element of the magnet is in precisely the same magnetic condition; the magnetisation being then called uniform. In the case of very long thin bar magnets, for example, the demagnetising influence of the ends is small; the general direction of the molecular magnetic axes being parallel to the axis of the bar throughout most of its length, and the lines of force appearing almost exclusively at the ends.

To form a model of a bar thus uniformly magnetised we may arrange end to end the little blocks described in § 95, so as to form straight chains of equal length, placed side by side.

The more molecular chains there are ending at the terminal faces of a uniformly magnetised bar, the more lines of force emerge into the surrounding field. Since we suppose every element of the bar to be in the same magnetic condition, the strength of pole m is proportional to the cross-section ω, and the same is true for every uniform longitudinal strip of the bar. Again, for any portion of the length of the magnet, the magnetic moment (§ 67) is greater the greater the length, for the ends of the molecular chains, which alone act as poles, are then further apart. A uniformly magnetised bar, as well as any smaller bar cut from it, has therefore a magnetic moment proportional to its cross-section ω, and to its length l, that is, proportional to its volume v.

97. **The toroid or anchor-ring.**—The completely uniform magnetisation of a bar is prevented by the disturbing effect of the ends. Here the lines of force begin to diverge

from one another, as do also the chains of molecular magnets. To obtain an approach to uniform magnetisation in a mass of iron or steel, we must make it in the form of a ring, or some such endless shape. We may then magnetise it so that no lines of force leave or enter it, the direction of magnetisation being everywhere tangential to the surface. When the ring is circular in form and of circular cross-section it is called an anchor ring or toroid (French *tore*=a ring). Since such a toroid neither emits nor absorbs lines of force, it is without external magnetic influence. In its interior also it may be shown that the magnetic force vanishes.

By the method of 'divided touch' (§ 19), we may produce a very fair approach to uniform magnetisation in such a ring. A more effective method of magnetisation will be described later on.

98. **The divided toroid.**—Even when the toroid is not quite complete, but is divided by a gap, we may assume the magnetisation to be uniform, provided that the breach of continuity is not too considerable. In that case the lines of force issuing from one face of the gap pass directly over to the other face, comparatively few of them spreading out into the surrounding region.

A nearly closed magnetised toroid, divided only by a narrow air-gap, we shall call a 'divided toroid.'

These two examples of approximately uniform magnetisation, the closed and the divided toroid, have important technical applications. Since they have no assignable poles, their properties cannot be deduced from the ordinary action at a distance theory.

99. **Magnetic moment per unit volume; intensity of magnetisation.**—For a uniformly magnetised body, or for any part of it, the magnetic moment divided by the volume has a perfectly determinate value, which depends upon the degree to which the body is magnetised. For the quotient in question depends on the number and strength of molecular magnets per unit volume, and the closeness with which their axes agree with one another in direction.

It is called the 'magnetic moment per unit volume,' or the 'intensity of magnetisation,' sometimes simply the 'magnetisation.' If m is the strength of pole of a uniformly magnetised bar whose cross-section is ω and length l, the intensity of magnetisation is

$$\mathfrak{I} = \frac{m\,l}{\omega\,l} = \frac{m}{\omega} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

Thus the magnetisation may also be defined as the strength of pole per unit area at the free ends. We have also from (6) the relation $m = \mathfrak{I}\,\omega$, which we shall find useful. Thus a bar which is uniformly magnetised with intensity $\mathfrak{I}$ emits at its north-seeking end $4\pi\mathfrak{I}\omega$ lines of force, that is $4\pi\mathfrak{I}$ per unit of area.

When a body is not uniformly magnetised, we may take an element so small that throughout its extent the departure from uniformity is not perceptible. The magnetic moment of this small element, divided by its volume is its intensity of magnetisation. For a long bar magnet, $\mathfrak{I}$ is constant throughout the indifferent region, but changes somewhat in value as we approach the ends.

100. **Dimensions of magnetisation.**—Since the magnetisation $\mathfrak{I}$ is expressible as a magnetic moment divided by a volume, its dimensions must be, by § 67

$$\frac{[L]^{\frac{5}{2}}[M]^{\frac{1}{2}}[T]^{-1}}{[L]^3} = [L]^{-\frac{1}{2}}[M]^{\frac{1}{2}}[T]^{-1}$$

the same as the dimensions of field intensity (§ 70).

B.—Induced magnetisation

101. **Behaviour of soft iron in the field; magnetisation by induction.**—We have already made use of soft iron to bring to more distant parts of the field the lines of force issuing from a magnet-pole. It may now be shown that the iron itself can acquire a certain intensity of magnetisation; though in this case the magnetic condition is only maintained so long as the external magnetic influence is present.

Experiment 42.—A bar of soft iron is held vertically in a stand, and over its upper extremity is laid a piece of stiff paper;

the pole of a strong steel magnet is then approached to the bar from above, the paper preventing actual contact between the two. (In this respect the experiment differs from that of § 18, p. 12.) If a vessel of iron filings is held close beneath the iron bar, magnetic properties will be manifested so long only as the bar is traversed by the lines of force of the permanent magnet; the magnetic condition is 'temporary.'

Experiment 43.—A strong bar magnet and a thin soft iron bar of equal length are placed end to end, the north pole of the magnet being in contact with the soft iron. A sheet of paper is then laid over the two, and the line-of-force diagram formed with iron filings. Lines of force are seen extending from one end to the other of the iron bar: they originate at the further end of the bar, and terminate at the end in contact with the magnet; the former end acting as a source, the latter as a sink. The phenomena observed are just what would result from the generation of a south pole at the nearer end of the iron bar, and a north pole at the further end. If the iron is brought into contact with the south pole of the magnet, exactly the opposite is observed.

Experiment 44.—In both dispositions of the magnet and iron bar, let the course of the lines of force in the neighbourhood of the iron be traced by means of the needle, fig. 7, p. 37, the positive direction of the lines being distinguished.

Experiment 45.—A thin bar of soft iron is put lying between the poles of two strong bar magnets, without touching them. By means of iron filings, the existence of poles at the ends of the bar may be rendered evident. The bar is then cut into short lengths without removing it from the field, and the separate pieces are arranged end to end with narrow air-spaces between. It will be found that there are poles at the ends of each piece, the lines of force passing on from one to the next as they would pass between same-way oriented bar magnets. On removing the permanent magnets, the polarity of the pieces of iron disappears.

When a piece of soft iron is introduced into the field of a magnet it disturbs the distribution of lines of force, one end of the piece becoming a source, the other a sink, so that the iron itself behaves as a magnet, which always turns towards the exciting poles, poles of opposite polarity. This phenomenon is called '*magnetisation by induction.*'

The magnetisation disappears as soon as the inducing magnet is removed, though usually a small degree of magnetisation persists; and this effect is called 'residual magnetism.' It is probable that the magnetism of natural magnets is a residual effect due to some previous magnetising agency.

Steel is also magnetisable by induction.

Experiment 46.—Two strong steel magnets and a steel knitting-needle are placed in a row with their adjacent ends nearly but not quite in contact. Tap the needle several times with a little wooden hammer, so as momentarily to loosen the mutual attachments of its molecules, and when removed from the field it will be found to be permanently magnetised.

Experiment 47.—Let an unmagnetised steel rod be held pointing in the direction of the earth's magnetic field, and let its upper end be struck several times with a hammer. By means of a sufficiently sensitive compass needle it may be shown that the two ends of the bar have acquired opposite polarities.

If we wish to determine whether a piece of iron or steel is magnetised, we must take account of the effects which may arise from induction. If the specimen is brought near to either pole of a sensitive compass needle, it collects together the lines of force which issue from the pole, and attraction takes place; unless the portion of the specimen held nearest to the pole of the needle is already a sufficiently strong pole of like name. It is only when one pole of the compass needle is repelled by some part of the piece of iron or steel that we can be quite sure that a pole already exists there, and that the specimen is in some degree magnetised.

102. **Magnetic screening.**—The important fact that the course of the lines of force may be modified by the effects of induced magnetisation, may be applied when we wish to concentrate the lines at some given part of the field, or to keep some part free from lines of force by means of a 'magnetic screen.' As examples we may here refer to several typical cases which are of great practical interest, and relate to the introduction of a soft iron ring into the field.

(*a*) *Course of the lines of force in a plane through the axis of symmetry of the ring.*—The ring is placed with its

plane vertical between the poles of an upright horse-shoe magnet.

A horse-shoe magnet, whose poles are about 8 or 10 cm. apart, is held in a wooden stand, with its two limbs pointing upwards. Over the ends of the magnet is fitted a board which has two holes cut to receive them, and between the holes is a longitudinal depression for the reception of the iron ring. The ring has the form of a short hollow cylinder, 4 or 5 cm. in diameter, and 2 or 8 cm. in height. The diameter of the ring is in the line joining the poles of the horse-shoe magnet. The course of the lines of force may be followed even within the ring, if a piece of paper of the proper size is passed through the ring and stuck down to the board, so as to continue the horizontal surface.

The lines of force from the two magnet-poles pass directly over to the ring, and enter its nearest parts. It gathers the lines of force together, so that even those which were originally more remote bend round to meet it.

The concentration of the lines of force by such an iron ring plays an important part in the working of dynamo machines.

(*b*) *Course of the lines of force in the principal plane of the ring.*

The ring is laid horizontally upon the board, fig. 31 giving a view as seen from above, a sheet of paper with a circular aperture being fitted over the ring. Within the ring a circular disc of paper is supported on a piece of cork, whose thickness is equal to the height of the magnet-poles above the surface of the board.

If we compare the line-of-force diagram (fig. 31) with fig. 3, which shows the disposition of the lines of force between the two poles when there is no ring, we see how the lines proceeding from the north pole pass over to the ring, bending round to it even from remote parts of the field, and there appearing to terminate. We know, however, that they travel through the substance of the ring, which, owing to its greater permeability, affords an easier passage than the surrounding medium. At the other side they emerge, and following a similar course, make their way to the south pole. There are two points of the ring's circumference (above and below in the figure) where the lines of force neither enter nor leave the iron, but run parallel to its boundary. They correspond to the two indifferent zones J, J of the magnetised ring.

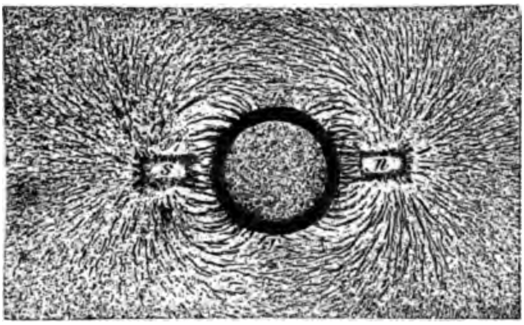

Fig. 31

In the space within the ring the iron filings are subject to no directing force, their distribution being quite irregular. Although there are strong magnet poles so near to it, this space is completely shut off from magnetic influences. This exemplifies a very important property of soft iron sheets: *they act as screens against magnetic forces.*

We can now understand the effect of a plate of soft iron in weakening magnetic influences, as found in experiment 27, § 33. The iron had gathered into itself the lines of force from the deflecting magnet, these lines entering the plate at one side of the rim, and emerging at the other side. Hence the effect on the movable magnet was very small.

103. **Screening of a magnetic needle by soft iron.**—The effect of soft iron in screening off disturbing magnetic influences, even when these are very considerable, is of such great importance that we will determine its amount in one case by direct measurement.

Experiment 48.—To the middle of a strongly magnetised strip of steel, about 3 cm. in length, is fastened a strip of drawing paper, whose ends are bent downwards, so as to lower the position of the centre of gravity. The steel strip is slightly indented at its centre, to enable it to be supported upon a needle-point, about which it turns freely; being kept from falling off by the paper attached to it. The needle point projects from a small wooden support which stands vertically upon a base. From a length of iron gas-piping about $3\frac{1}{2}$ cm. external diameter a piece about 10 cm. long is cut, and well annealed.

Unlike poles of two strong bar magnets are arranged at such a distance apart that the piece of iron piping can be easily interposed between them; the magnetised strip, supported on the needle point, being between the two magnets, and in line with them. Before the iron piping is introduced let N be the number of swings made in unit time by the needle, so that (§ 75) the strength of the approximately uniform field between the two poles is proportional to N^2.

The iron tube is now lifted over the needle and lowered carefully without disturbing the adjustments, the lower ends of the paper being seen below the lower edge of the tube. The oscillations of the needle are now much slower, and if n is the number now executed in unit time, the former field-intensity is to the present one as $N^2 : n^2$. The strip of paper attached to the needle renders the oscillations visible to a large audience. It is not advisable to suspend the needle by a thread, as it would then be easily attracted to one side of the iron cylinder, and there remain in contact.

When fine magnetic measurements have to be made, a protecting screen of soft iron is often of great use, to shut off the magnetic influence of the strong currents used for technical purposes, whether these act directly or through the effect of leakage currents in the ground, as is often the case in cities.

104. **Conception of magnetic polarisation.**—Faraday showed that not only steel and iron, but all substances, are to some extent magnetisable, though in most cases very strong magnetising forces are required to produce a perceptible degree of magnetisation. We may thus consider all media through which a magnetic field extends to have chains of molecules extending through them. These media will accordingly be in a certain condition of stress, since the opposite polarities excited in the smallest particles react

upon one another. We speak, therefore, of a certain condition of magnetic polarisation which obtains in steel, soft iron, or any other substance exposed to magnetic influences. The question arises whether we can so conceive of this condition of stress in a magnetically polarised medium, as to obtain an explanation of the tension and pressure which we have already shown to exist along and across the lines of force respectively. We proceed to consider two elements lying side by side and polarised in the same sense.

Experiment 49.—Two thin soft iron rods, only a few centimetres in length, have each a loop at one end, to which a long thin silk thread is attached. The rods are suspended by means of the threads, so that they hang side by side at the same height, and a pole of a strong bar magnet is then brought near to them. They are seen to move away from one another. Through each of the rods passes a part of the flux of force which issues from the pole of the magnet, the upper ends of the rods thus acquiring the same polarity as the exciting pole, and their lower ends the contrary polarity. The two pairs of like poles, above and below, cause a repulsion between the two rods—that is to say, there is a pressure between them, and they accordingly separate until the restoring force due to gravity is great enough to preserve equilibrium.

If we conceive of a medium as made up of chains of such elements, which are joined end to end, the chains lying side by side, the separate chains must similarly exert lateral pressure on one another. But these chains indicate the course of the lines of force through the medium. From the polarisation of the separate elements, therefore, there arises a pressure across the direction of the lines of force.

We may similarly infer the existence of a tension along the lines of force. For we know that lines of force proceeding from the pole of the bar magnet pass along the small iron bars from one pole to the other, and since two poles of the same bar are of opposite polarity, they necessarily attract one another. When the chains of iron filings between two strong unlike poles continue to crowd more and more into the middle of the field as the paper supporting them is tapped, some part of the effect may be attributed to the *shortening of the chains of polarised particles, owing to the tension in the direction of their length.*

This explanation is, however, only apparent, since we have assumed the existence of that tension along the lines of force which we set out to explain. Thus the assumption of a polarisation is not by itself sufficient to afford a mechanical explanation of the pressures and tensions in the field. It has therefore been necessary to subject the phenomena to a further analysis, by means of mechanical modes of representation which will be described later.

105. **Paramagnetic and diamagnetic substances.**—All substances do not behave like soft iron towards the lines of force, collecting the lines more closely together, so that fewer are left to traverse the surrounding air. We have already (§ 14) ascribed this phenomenon to the greater permeability of soft iron to the lines of force. Certain substances, such as bismuth for example, have a magnetic permeability smaller than that of air. Lines of force proceeding through the air give way as they approach such a substance, bending round it so that fewer pass through than if the same space had been filled with air. Media which behave in this manner are called 'diamagnetic,' to distinguish them from paramagnetic media which behave like soft iron.

Models of paramagnetic and diamagnetic bodies in uniform fields.—We will now describe two models, illustrating the greater permeability of paramagnetic substances, the lesser permeability of the diamagnetic, and the corresponding changes produced in the distribution of lines of force in a uniform field.

Along the shorter sides of two rectangular boards, 48 cm. long and 24 cm. broad (fig. 32), laths 2 cm. in thickness are fastened, one lath on each board being painted red and the other blue, to represent the north and south poles of a magnet. In the middle of each board is fastened a wooden block about 15 cm. long and 10 cm. broad. Through holes in the marginal laths wires are passed, and to these arrow-heads are attached, pointing from the north towards the south pole. The wires are to represent the lines of force. The outermost wires pass directly across from one lath to the other, but in the middle of the series there are some short wires which are joined to the wooden blocks. All the wires are bent in the manner indicated in the figure.

From the disposition of lines of force in fig. 32 we conclude:

(*a*) That in the case of a paramagnetic substance such as iron (*Fe*), the lines of force issuing from the north pole are collected together by the body lying in the field, passing to a great extent through the body, and diverging again after leaving it. The ends of the body facing the north and south poles of the magnet become a sink and a source respectively, that is, of opposite polarity. If the body is free to turn in the field, the tension along the lines of force tends to set it parallel to the direction *n*—*s* in the figure: 'axial' or 'polar setting.'

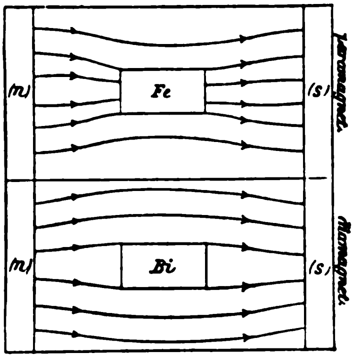

FIG. 32

(*b*) That in the case of a diamagnetic substance such as bismuth (*Bi*) the lines of force issuing from the north pole are diverted on encountering the nearer end of the body, just as if there were a pole of like name there. They pass mostly through the surrounding medium, which they can traverse more easily. At the other side of the diamagnetic body the lines of force once more converge, just as if they were sucked in by the presence of a sink there.

The end of the body facing the north-seeking pole of the magnet is a source, and therefore also north-seeking, the end opposite the south-seeking pole being a sink, that is, south-seeking. When the diamagnetic body is free to turn, the pressure across the direction of the lines of force will tend to set it perpendicular to the direction *n—s*: 'equatorial setting.'

The polar setting of paramagnetic and equatorial setting of diamagnetic substances may be demonstrated by preparing short bars of suitable materials, and suspending them by thin cocoon fibres. Very strong magnetic fields are necessary for the experiment, and the way in which these may be obtained will be described in Section II.

The behaviour of the two classes of substances towards the lines of force may be illustrated from the following example in fluid motion. If the bed of a straight broad flat stream is thickly overgrown with rushes, so that the flow of the water is considerably resisted, and if the rushes are cut away from a rectangular patch in the middle, so that within that patch there is no obstacle to the flow, the stream lines will be gathered together and pass more thickly through this place of greater permeability, just as the magnetic lines of force do through soft iron. If, however, there are no obstacles in the bed of the stream except a patch of rushes in the middle, the stream lines bend round so as to avoid the place of greater resistance, just as the lines of force do in the case of bismuth.

If the permeability of the surrounding medium is changed, a paramagnetic body may be thereby converted into a diamagnetic one, or *vice versâ*; so that one and the same body may set axially or equatorially between the poles of a magnet, according to the nature of the surrounding medium.

C.—Flux of magnetic induction

106. **Magnetic induction.**—In the interior of a magnetised body the lines of force are propagated from molecule to molecule. If the magnetisation is uniform, it is to be measured by the intensity $\mathfrak{J}$ (§ 99). Suppose the body to be placed in a magnetic field of strength $\mathfrak{H}$, measured by the number of lines of force per square centimetre of cross section; this strength of field is then added to that already existing within the body.

We shall later on become acquainted with very simple methods of superposing two or more magnetic fields.

The magnetic condition is determined by the total number of lines of force per unit of cross-sectional area, the important magnitude thus measured being called the 'magnetic induction.' To show its significance as clearly as possible, we shall make use of a simple example. Let there be a toroid, uniformly magnetised with intensity $\mathfrak{J}$, and let this toroid be cut through at one place, so as to leave two free ends separated by an air-gap. We shall suppose the gap to be so narrow as not to disturb the uniformity of magnetisation. Then in accordance with § 99, there will be $4\pi\mathfrak{J}$ lines of force passing across each square centimetre of the gap, and these continue their course to the same number through the substance of the toroid. Upon this system of lines of force we have to superpose whatever system may be due to external causes. In the latter let there be $\mathfrak{H}$ lines of force per cm.^{2}, so that $\mathfrak{H}$ is the strength of the external field in absolute measure.

Then within the air gap there will be $4\pi\mathfrak{J} + \mathfrak{H}$ lines of force per cm.^{2}, and the same lines continue their course uninterrupted through the substance of the toroid. The magnetic condition is characterised by the magnetic induction

$$\mathfrak{B} = 4\pi\mathfrak{J} + \mathfrak{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Since all substances placed in a magnetic field become magnetised to some intensity $\mathfrak{J}$, we see that $\mathfrak{B}$ is always to be distinguished from $\mathfrak{H}$; when we have confined our attention to the strength of field $\mathfrak{H}$, we have tacitly assumed $\mathfrak{J} = 0$; this being justified by the small magnetisability of air, with which medium we were usually concerned.

107. **Dimensions of the magnetic induction.**—Since the numerical factor 4π is without dimensions, it follows that $\mathfrak{B}$ is the sum of two quantities, each of which has the dimensions cm.$^{-\frac{1}{2}}$ gr.$^{\frac{1}{2}}$ sec.$^{-1}$ (§ 100 and § 70). In other words, magnetic induction, intensity of magnetisation, and strength of field are all of the same dimensions. When we write

$\mathfrak{B}=4\pi\mathfrak{J}+\mathfrak{H}$, we assume that the lines of force arising from $\mathfrak{J}$ are coincident in direction with those of the magnetic field $\mathfrak{H}$ due to external causes. If instead the two systems of lines ran in opposite directions, $\mathfrak{H}$ would have to be subtracted. The induction $\mathfrak{B}$ is a directed quantity like the field-intensity $\mathfrak{H}$; that is, it has a determinate direction as well as a determinate magnitude.

108. **Tubes of induction and flux of induction.**—A line of induction is one which at each point satisfies the condition that its tangent is in the same direction as the magnetic induction. As in the case of lines of force, the number of lines of induction which cut (normally) through unit area, measures the strength of the induction. A space enclosed by a tubular surface, made up entirely of lines of induction, is called a tube of induction.

There is also a quantity analogous to the flux of force, namely, the flux of induction I, which for each tube is the product of the induction $\mathfrak{B}$ and the cross-section ω of the tube; that is, $I=\mathfrak{B}\omega$. Since, in accordance with § 106, it is always with the induction that we are concerned, it is really only tubes of induction that can be drawn, and all the diagrams of Chapter V. are essentially diagrams of the flux of induction. It is only when we are dealing with a medium for which $\mathfrak{B}=\mathfrak{H}$ that we can identify the flux of induction with the flux of force; but since this condition is very nearly fulfilled in the case of such a feebly magnetisable substance as air, we were able to speak of our diagrams as relating to the flux of force. Throughout every tube of induction, the flux of induction I is always absolutely constant.

109. **Measure of magnetic permeability.**—The magnetic induction $\mathfrak{B}$ furnishes a convenient means of expressing numerically the value of the magnetic permeability. Since $\mathfrak{B}$ denotes the total number of lines of force which cut through each unit of cross-sectional area within a body placed in a magnetic field, the ratio

$$\mu=\mathfrak{B}/\mathfrak{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

furnishes a measure of the extent to which the body has collected together the lines of force of the field $\mathfrak{H}$, and caused them to pass more thickly through it. This effect of the body we attribute to its greater permeability. It is accordingly to be measured by the ratio of the actual number of lines of force $\mathfrak{B}$ per unit area, to the number $\mathfrak{H}$ which would have been present in the absence of the body from the field. Since $\mathfrak{B}$ and $\mathfrak{H}$ are of the same dimensions, the permeability μ is without dimensions, in other words it is a pure number. If we put $\mu=1$ for air (the medium in which we generally work), then

for all paramagnetic substances $\mu > 1$,
for all diamagnetic substances $\mu < 1$.

For good soft wrought iron μ may reach as high a value as 2,000 or 3,000, while for bismuth $\mu=0{\cdot}99982$. In a medium whose permeability is unity, the flux of induction becomes identical with the flux of force, and the tubes of induction with the tubes of force.

110. **Susceptibility.**—In the last paragraph we considered the ratio of the induction to the field-intensity; but for some purposes it is more convenient to measure the ratio of the intensity of magnetisation $\mathfrak{I}$ to the field-intensity $\mathfrak{H}$. This latter ratio is called the magnetic susceptibility κ, so that

$$\kappa=\mathfrak{I}/\mathfrak{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In accordance with the assumption which we have made, $\kappa=0$ for air, while

for paramagnetic substances κ is positive ($\kappa > 0$),
for diamagnetic substances κ is negative ($\kappa < 0$).

In the case of soft iron, κ lies between 200 and 300. Since $\mathfrak{I}$ and $\mathfrak{H}$ are of the same dimensions, κ is like μ a pure number.

111. **Relation between permeability and susceptibility.** Since the permeability $\mu=\mathfrak{B}/\mathfrak{H}$, the susceptibility $\kappa=\mathfrak{I}/\mathfrak{H}$, and $\mathfrak{B}=4\pi\mathfrak{I}+\mathfrak{H}$, we have

$$\mu\mathfrak{H}=4\pi\kappa\mathfrak{H}+\mathfrak{H}$$

$$\text{or } \mu=1+4\pi\kappa \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\text{whence also } \kappa=\frac{\mu-1}{4\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The magnetic character of a substance is determined by either of the quantities μ or κ, and the last written equations enable us, when one is given, to determine the other.

The two quantities μ and κ are not in general constant for any given substance, but are functions of the field intensity $\mathfrak{H}$, so that the permeability and susceptibility of a substance are different according as it is placed in a weak field or a strong one.

D.—The fundamental property of the magnetic induction

112. **Magnetic circuits.**—When we defined the magnetic induction by the relation $\mathfrak{B}=4\pi\mathfrak{J}+\mathfrak{H}$, we were dealing with a definite position in the field, where the magnetic conditions due respectively to the internal lines of force and the external field have to be superposed. In many cases, however, especially in those which are of the greatest practical importance, it is more convenient to consider as a whole the system of tubes of induction, which bend round so as to form closed circuits. This is possible, since we know of a quantity which remains constant along a tube of induction, and whose value is characteristic of the tube. This quantity is the flux of induction $I=\mathfrak{B}\omega$. The lines of induction of a magnet pass through the interior as well as through the surrounding field, any bundle of the lines which we may consider separately passing from the surrounding space into the magnet and out again into the surrounding space without any change in their number. In a medium of permeability 1 we found that the flux of force along each tube was constant so long as no sources and sinks were encountered. The constancy of the flux of induction along each tube of induction is absolute, and holds good under all conditions. It is the law of the conservation of the flux of induction with which we are concerned when the field

extends through media for which the permeability μ has different values, so that at the surfaces which separate such media from one another there are sources and sinks of lines of force. When tubes of induction pass from a medium of small permeability (such as air) into one of higher permeability (such as iron or steel) they become more closely crowded together, so that the induction $\mathfrak{B}$ attains a higher value; when, leaving this medium, they enter one which is feebly paramagnetic or is diamagnetic, they diverge again correspondingly. Each tube of induction, taken in its entirety, is an annular portion of space, possessing the property that through every cross-section of it the flux of induction I has the same value. A collection of tubes of induction lying side by side, and occupying a finite portion of space, constitutes what is called a '*magnetic circuit.*'

A horse-shoe magnet with its keeper attached, closed or divided toroids, all furnish examples of magnetic circuits. In the last-mentioned case the lines of force which traverse the air-gap are continued through the substance of the toroid, where the high value of the induction is largely due to the high magnetisation. As other examples of magnetic circuits may be mentioned the system of magnets in a dynamo machine and the iron cores of transformers.

113. **Magneto-motive force.**—This quantity may be taken as a measure of the total magnetising influence along a closed tube of induction. If at a place where the field-intensity is $\mathfrak{H}$ units, we move the unit pole through a distance s in the direction of the magnetic force, the work done by magnetic actions upon the pole will be $\mathfrak{H}s$. If the unit pole is moved through a further distance along the tube of induction, a further amount of work will be done, and this must be added to the former amount. If the pole has accomplished a complete circuit of the tube, returning once more to its starting point, the total work done is a quantity characteristic of the tube of induction. It has been called the 'magneto-motive force,' M, and is a quantity of work referred to unit pole, that is it has the dimensions (work/strength of pole) or M has the dimensions:

$$[L]\,[M]\,[T]^{-2}\,[L] \div [L]^{\frac{3}{2}}\,[M]^{\frac{1}{2}}\,[T]^{-1} = [L]^{\frac{1}{2}}\,[M]^{\frac{1}{2}}\,[T]^{-1}$$

Since we are concerned with the quantity of work expended upon a unit pole, and not with the force exerted upon it, the name magneto-motive 'force' is not very well chosen; in this respect, however, it is on the same footing as the term electromotive force applied to the corresponding electrical quantity.

114. **Magnetic resistance.**—The magnetic resistance ρ is defined as the reciprocal of the magnetic permeability or 'conductivity.' Accordingly we write:

$$\rho=1/\mu \text{ or } \rho=\mathfrak{H}/\mathfrak{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The greater the magnetic resistance of a substance, the smaller is its permeability; for air, paper, brass, ρ is large, while for soft iron it is small. The following roughly quantitative experiment is designed to show the marked effect produced upon a magnetic circuit by the interposition of even a thin layer of a substance whose magnetic resistance is high.

Experiment 50.—To the poles n and s of a small horse-shoe magnet (fig. 33) is applied a soft iron keeper, semicircular in form and having a loop attached to it, from which a scale pan may be hung by means of a hook. In addition to this undivided keeper, there is another of the same shape and weight, but having only its outer portions l_1 and l_2 (fig. 33) made of iron. In the middle, between l_1 and l_2, a piece of brass m is interposed, and the brass ring h is held in position by two screws s_1, s_2, so that it helps to hold the several portions of the keepers together. We have thus a keeper m in which a body m of high magnetic resistance ρ is interposed in the course of the lines of force which traverse the strongly paramagnetic portions l_1 and l_2. We now proceed to determine the pull required to detach the keeper from the magnet (a) when the ordinary keeper is used; (b) when the divided keeper is used; (c) when a sheet of paper is interposed between the iron keeper and the ends of the magnet: (d) using the divided keeper under the same conditions.

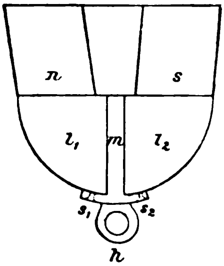

FIG. 33

Numerical example:

Lifting power of a horse-shoe magnet

(*a*) 2·220 kilogr. + 20 grams (weight of keeper and scale pan);
(*b*) 0·580 kilogr., only about $\frac{1}{4}$ of the preceding.
(*c*) 1·100 kilogr., about half the value in (*a*).
(*d*) 0·190 kilogr., only $\frac{1}{12}$ of the full lifting power.

The lifting power is not proportional to the number of lines of force; but the experiment shows the great decrease in the flux of induction produced by an increase in the resistance of the circuit.

When some portion of a magnetic circuit is of small permeability, the resulting resistance is greater the longer the portion in question, and less the greater its cross-section; for when the cross-section is large, the same flux of induction can be spread over a greater area; the portion difficult to magnetise being then less strongly magnetised than if the whole system of lines of force were concentrated on a very small cross-sectional area. If ρ_0 denotes a factor characteristic of the material, l the length of the piece in question, and ω its cross-section, we can write for the magnetic resistance

$$w = \rho_0 \frac{l}{\omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The factor ρ_0 is the resistance of a block of the substance, whose length is 1 cm. and cross-section 1 cm.^{2} It is called the 'specific magnetic resistance,' and is identical with ρ as defined by (12).

If a magnetic circuit consists of a number of successive parts, whose separate resistances are w_1, w_2, w_3 . . . . the resistance of the entire circuit is

$$W = w_1 + w_2 + w_3 + \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

115. **Fundamental law of the magnetic circuit.**—If M is the magneto-motive force around a circuit, made up of tubes of induction, and having the magnetic resistance W, the flux of induction I is given by the ratio of M to W.

The flux of induction I, that is the number of lines of *induction within a given tube of induction, is equal to the magneto-motive force M, divided by the total magnetic resistance W which the lines of induction encounter*; that is

$$I=\frac{M}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

This law was formulated in 1886 by J. and E. HOPKINSON and G. KAPP. It is analogous to Ohm's law in electricity. But whereas in the latter case the resistance is a true constant, the magnetic resistance $\rho=1/\mu$ must, in accordance with § 111, be regarded as a function of the field intensity.

E.—Energy in the magnetic field

116. Work of magnetisation.—In magnetising a piece of steel, a certain amount of energy must be expended, such as we measure in the simplest case by the product of a force and a displacement. If the method employed is that of rubbing the bar to be magnetised with a permanent magnet, we shall have to consider the mutual action of two systems of lines of force: one arising from the permanent magnet, the other from the newly acquired poles of the piece of steel which we are magnetising. Supposing that during the operation the centre of gravity of the moving bar remains always at the same height, and neglecting the frictional forces which have to be overcome, there is still a certain amount of work to be done against the tensions and pressures which accompany the lines of force. If the north pole is being used to produce the magnetisation, it has to be moved away from that end of the steel where a south pole has been produced, and towards the north-seeking end; to accomplish which, work must be expended.

The process of magnetising a bar of steel consists in imparting to it a certain quantity of energy in a special form.

The work which must be expended in the process is called the work of magnetisation; no change is produced in the mass of the bar, as determined by the balance (§ 20).

In accordance with the principle of the conservation of energy, no work is lost during the transformation. So long as it is not further changed into other forms, the transformed energy remains permanently in the magnetised bar.

If a body be magnetised alternately in opposite senses, after some time it becomes perceptibly heated, a considerable portion of the work of magnetisation being transformed into heat energy.

117. **Localisation of the energy of the field.**—A portion of the work expended in magnetisation will be consumed in changing the disposition of the molecules, uniting them to form those chains of molecules which distinguish a magnetised from an unmagnetised body. Another portion of the work of magnetisation is stored in the field surrounding the magnet. The space around a magnet-pole is in a certain sense impenetrable to the field of another pole of like name. *The impenetrability implies the existence of a volume-distribution of energy.* We must conceive of the magnetic energy as residing in the separate volume-elements of the magnetic field.

The pressures and tensions to which this form of energy is related may be made directly evident (QUINCKE). The existence of the field changes the optical behaviour of the medium, which is evidence that this same special form of energy resides in the field itself.

118. **Mechanical example of the localisation of energy.**—To one end of an elastic wire is attached a load p, considerable in comparison with the weight of the wire. The wire, being fixed at the other end, will be extended by a length l, proportional to the weight p; so that we may write $p=\epsilon l$. The factor ϵ is constant within wide limits; it depends on the length, cross-section and material of the wire, that is, on the special properties of the medium in which energy is to be stored. To extend the wire by a further amount dl requires the expenditure of work dA = force × displacement $= p.dl = \epsilon l.dl$, which depends on l as well as

on dl. The quantity of work dA required to produce an additional extension dl is accordingly greater, the greater the force p already applied. Corresponding to any given extension of the wire, the amount of work which has been expended and is stored up in the wire, is proportional to the square of l. For on integrating the expression dA, we obtain

$$A = \tfrac{1}{2}\epsilon l^2;$$

the wire having no energy of extension when $l = 0$. Hence l determines the amount of elastic energy residing in the wire.

The value for dA may be resolved into factors in two ways:

$$dA = \epsilon l . dl \text{ and } dA = l . d(\epsilon l).$$

And similarly, for A we have

$$A = \epsilon \frac{l^2}{2} \text{ and } A = (\epsilon l)\,\frac{l}{2}.$$

Each expression for the energy is the product of two factors; the form $A = \epsilon l . l/2$ having the peculiarity that the two factors are not independent of one another.

119. **Measure of the energy of the magnetic field.**—That the accumulation of magnetic energy in a field must be accompanied by effects somewhat similar to those in our illustrative example (§ 118) is made clear to us when we consider the lines of force, the deformations which they suffer when a pole (for example a unit pole) is introduced into the field, and the pressures and tensions which we know to exist. Since a pole in a magnetic field experiences a force which is greater the more closely the lines of force are packed, that is, the greater the field-intensity $\mathfrak{H}$, it follows that the introduction of new lines of force, and with them new energy, into the field, is more difficult the higher the value which the field-intensity has already reached.

Let $\mathfrak{H}$ be the field-intensity and $\mathfrak{B}$ the magnetic induction at some given point, and let us consider a surrounding region (called the region R) so small that nowhere within it does the intensity differ appreciably from $\mathfrak{H}$ or the induction from $\mathfrak{B}$. We shall have also $\mathfrak{B}=\mu\mathfrak{H}$, where μ may or may not be $=1$. Now suppose that we have a magnet in the form of a divided toroid, the width of the air-gap being l and its area F. Let us further suppose that the degree of magnetisation of the toroid is very small, so that the value of the magnetic induction $\delta\mathfrak{B}$ across the air-gap is also very small. The strength of pole m of the toroid is given by

$$4\pi m=Fd\mathfrak{B} \text{ or } m=F\frac{\delta\mathfrak{B}}{4\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and we shall suppose the steel to be 'rigidly' magnetised, so that m suffers no change from the action of external fields.

Now introduce the toroid into the field to be examined, the air-gap being made to lie within the region R with its faces perpendicular to the direction of $\mathfrak{H}$ and $\mathfrak{B}$, and so that $\mathfrak{B}$ and $\delta\mathfrak{B}$ are opposed to one another in direction. Thus though the volume Fl of the air-gap the induction is now $\mathfrak{B}-\delta\mathfrak{B}$.

Next let the position of the toroid be reversed in the field, in such a manner that each face of the air-gap occupies the position which was just now occupied by the other face. The induction through the air-gap thus becomes $\mathfrak{B}+\delta\mathfrak{B}$.

We must now find how much energy has been expended in changing the magnetic induction from $\mathfrak{B}-\delta\mathfrak{B}$ to $\mathfrak{B}+\delta\mathfrak{B}$, through a volume Fl, the mean value of the field-intensity being $\mathfrak{H}$. This energy must be equal to the work expended in turning the toroid round against forces due to the surrounding field, an amount of work which is easily calculated. For the nett result of the operation is the displacement of a *north* pole of strength $+m$ through a length l *against* the direction of $\mathfrak{H}$, and of a *south* pole of strength $-m$ through an equal length l *in* the direction of $\mathfrak{H}$. Both these displacements demand the expenditure of positive work, the total amount of which for the two poles is

$$\begin{aligned}\delta W &= 2\,m\;\;l \\ &= Fl.\mathfrak{H}.\frac{2\delta\mathfrak{B}}{4\pi}\end{aligned}$$

(on substituting for m the value given by (15)).

The last expression written is the product of three factors: the volume of the region affected by the operation, the value of the field-intensity for this region, and the increase $2\delta\mathfrak{B}$ of the magnetic induction divided by 4π. Hence if A is the magnetic energy *per unit volume* at a place where the field-intensity is $\mathfrak{B}$ and induction $\mathfrak{B}$, and if A becomes changed to $A+dA$ when $\mathfrak{B}$ is changed to $\mathfrak{H}+d\mathfrak{B}$, we shall have

$$dA=\frac{1}{4\pi}\mathfrak{H}\,d\mathfrak{B}=\frac{\mu}{4\pi}\mathfrak{H}\,d\,\mathfrak{H} \quad . \; . \; . \; . \; . \; . \; . \quad (16)$$

The value of A may be deduced from this by integration, supposing μ to be a constant; thus

$$A=\frac{\mu}{8\pi}\mathfrak{H}^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (17)$$

or

$$A=\frac{1}{8\pi}\mathfrak{B}\mathfrak{H}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

If we are dealing with air, the factor μ becomes $=1$, and may be omitted from all the equations.

The expressions for the forms of energy considered here and in § 118 are similar to those for the kinetic energy of a mass m, moving in a determinate direction with velocity v:

$$K=\frac{mv^2}{2} \text{ or } K=mv\frac{v}{2}.$$

The analogy, however, only extends to the form of the expressions. The energy of a stretched body is not spoken of as kinetic, and it is not to be assumed without further inquiry that the energy of a magnetic field is of the ordinary mechanical kind.

[In this paragraph (§ 119) a slight departure has been made from the treatment in the original text.]

CHAPTER VII

DYNAMICAL THEORIES OF FIELDS OF FORCE

When a pole is placed in a magnetic field, it experiences a force of determinate magnitude acting in a determinate direction. We have already (§ 31) spoken of the magnetic force as a vector or directed quantity; a magnetic field may be said to be determined by an infinity of such vectors. The question now arises: Can we not discover the nature of those conditions, vectorial in character, which exist at each point of a magnetised medium, and are there not, in particular, some simple types of motion which possess the same character? The latter question is suggested by the development which has taken place in our understanding of other physical phenomena, such as light and heat; the older hypotheses which sought an explanation in the presence of special fluids or corpuscles, having given way to the doctrine of a special kind of motion. The kinetic theories and the mechanical theories connected with them have acquired a great importance; and we will accordingly give an account of the most important which have been formulated in connection with our subject. In doing this, we shall have to depart from a mere description of actual phenomena, and consider points of a more or less conjectural kind.

A.—Symmetrical properties of magnetic fields

In our endeavour to explain magnetic phenomena by means of known mechanical motions, we are guided by a number of fundamental properties, which leave no doubt as to the description of motion to be chosen. We have to proceed somewhat as we do in mineralogy, when we infer the structure of a crystal from its observed properties. There are also some properties of a magnetic field which imply a certain symmetry in the medium which is the seat of the field; and the conditions obtaining in the separate elements of volume must correspond to these symmetrical relations (CURIE).

120. **Axial character of the magnetic lines of force.**—The simplest kind of vector quantity is the displacement or translation of a small body from a point in space, through a determinate distance in a determinate direction to some other point. It is with vectors of this kind that we are concerned in all phenomena of flow, and we may conceive of the directional properties of a magnetic field as being brought about by the flow of something along the lines of force. We have already made frequent use of terms which are based on this conception, though at the same time we have pointed out that this flow of the lines of force is only a mode of representation. There are certain phenomena which show that we have not to deal with this simplest form of vector, but with vectors of a different kind. For example, there is the rotation of the plane of polarisation produced in a beam of plane-polarised light which is made to pass through a magnetic field along the direction of the lines of force (FARADAY). From this observation, as early as 1856, Sir WILLIAM THOMSON (Lord KELVIN) arrived at the conclusion that the conditions obtaining along a line of force are kinematically comparable with those along an axis of rotation. And in fact rotation about an axis is a phenomenon of a vectorial nature. The direction of the vector is the direction of the axis of rotation, the positive direction of the axis being determined in accordance with some fixed convention. Thus we may agree that a rotation in the given sense, accompanied by a translation along the positive direction of the axis of that rotation, shall constitute a right-handed screw-motion; or we may base our definition on the left-handed screw. Finally, the rotational velocity may be represented by setting off an appropriate number

of units of length along the axis of rotation. *Magnetic forces possess the same kind of symmetry as an angular velocity.* If we suppose the region with which we are concerned to be 'perverted' by reflection in a plane mirror, rotational movements will be affected differently from simple translations, whose directions we indicate by means of arrows. It will be shown that the same holds good for magnetic fields, from a consideration of the special forms described in Section II. F. Koláček, to whom we owe the very appropriate term 'axial character of the magnetic lines of force,' has quoted Hall's phenomenon as proving that there is something which rotates about these lines as axes.

121. **Rotational vectors.**—If we conceive of magnetic phenomena as due to some kind of motion which is rotational in character, there must be an invariable relation between the direction of rotation and the direction in which the lines of force are reckoned positively. We have already (§ 28) given a rule for determining the positive sense along a line of force, and we must now agree upon a similar rule for the corresponding rotational sense.

The rotation which we conceive to take place about a line of force would appear to be in the direction of clock-hands to an eye looking along the line in the positive sense, that is, from the north pole towards the south pole.

The propriety of this convention will appear in Section II.

It will be well to construct a small model (fig. 34), so as to be always able to recall easily the rule just given.

The cylindrical rod A A carries a spike S at its upper end, and at its middle is a wooden disc H to whose circumference two tin arrows P_1, P_2 are fastened. On looking along AA towards the spike S (direction of lines of force) we see the lower side of the disc, whose rotation, suggested by P_1 and P_2, is in the direction of the hands of a clock.

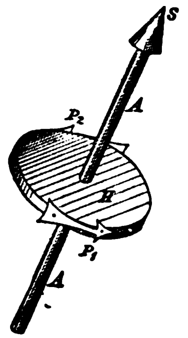

FIG. 34

The compound motion is similar to that of a corkscrew which is being screwed upwards through a cork, or of any other right-handed screw.

122. Principle of the superposition of small motions.—To determine the result of superposing two magnetic fields, for example those due to two bar magnets, we need to make use of a very simple principle, which is of the widest application in physics, and may be stated as follows for the present case:

If at some given position in space one field alone would give rise to a determinate rotational motion, and another field to another such motion, the resultant motion of the medium, which is the seat of the two fields, is to be found by compounding the two separate motions together.

A similar principle is applicable to the phenomena of acoustics, of optics and of wave-motion in general: its purport is that two or more independent systems of waves can coexist in the same medium.

If at any point the two rotational vectors are coincident in direction, their values must be simply added together to give the value of their resultant. In other cases the resultant, which is of course determinate in direction and magnitude, is to be determined by means of the following simple construction.

123. Composition of rotational motions; the parallelogram of rotational vectors.—Since every rotational vector may be represented in direction and magnitude by a straight line drawn from a point, the two rotational vectors which we have to compound will be represented by two straight lines drawn from a point in the field. If then a parallelogram be constructed having these two straight lines for adjacent sides, the resultant of the two rotational vectors will be represented by that diagonal of the parallelogram which is drawn from the point in question. It should be observed that when we speak of a line as being drawn *from* a point in a field, this is intended to indicate the positive direction along the line; the negative direction being from the further extremity of the line towards this point. In fig. 35, if q_1 is a rotational vector due to one magnetic field, and q_2 is a similar vector at the same point, due to a second field superposed on the first, Ω is the diagonal of the parallelogram whose adjacent sides are q_1 and q_2. This diagonal is determinate in direction and magnitude, and may therefore be taken to represent a rotational vector of the same kind as q_1 or q_2. It represents, in fact, a rotational motion which is the resultant of the separate motions q_1 and q_2.

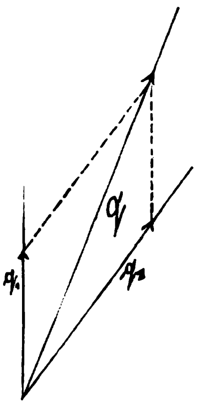

FIG. 35

Very various proofs of this kinematical theorem are to be found in elementary text-books. The reader may most easily satisfy himself of its correctness by considering any one point of a rigid body, and following its displacement as the body is rotated through small angles about the two axes successively; the small angles being proportional respectively to q_1 and q_2.

If the two vectors o_1 and o_2 are coincident in direction, the diagonal Ω is equal in magnitude to their sum; if on the other hand they are in exactly opposite directions, Ω is equal to their difference. Hence the following simple rule:

Where lines of force in the same direction are superposed upon one another, the rotatory motion is strengthened; where oppositely directed lines are superposed, the motion is diminished. If the lines of force belonging to the two original systems are inclined at an angle to one another, they must be compounded by the parallelogram of rotatory vectors, which leads to the rule for the combination of fields of force so frequently applied in Chapter V.

124. **Conception of a permanent motion.**—If each line of force is really an axis about which a rotatory motion is taking place, there might seem to be some difficulty in explaining the fact that this motion continues undiminished so long as the corresponding magnetic force retains its value

unaltered. For even bar magnets, with free ends from which lines of force are given off, lose their magnetism but slowly, while in closed magnetic circuits (such as that of a horseshoe magnet with a keeper) the intensity of magnetisation may remain almost constant for years. Here then rotatory motion must be maintained without appreciable loss of velocity. This would appear to contradict our hypothesis, since every observable motion of the kind which we postulate must ultimately be completely annihilated by friction, unless some means were from time to time used to excite it afresh. This, however, constitutes no real ground of objection against the existence of motion in the field, provided the special character of that motion is taken into account. It is a matter of every-day experience that the kinetic energy of finite bodies in motion becomes transformed by frictional resistances into energy of other forms, the most important of which is heat, and so that the velocity on which the kinetic energy depends is correspondingly diminished.

In the mechanical theory of heat, thermal phenomena are referred to motions of the smallest particles, or molecules, of matter. But though we may constantly observe the transformation of the energy of motion of appreciable masses into heat by friction, no similar transformation takes place in the world of molecules; we may always conceive of the molecular movements as unaccompanied by friction without finding anything in experience to contradict our supposition. The rotatory movements of MAXWELL are supposed to actuate the smallest particles, which we may suppose to be affected with permanent motions about axes, frictional influences being absent.

RICHARZ, taking as his starting point the phenomena of electrical atomic charges, has recently succeeded in explaining the magnetic properties of iron by means of such rotatory motions. The diffiiculty of conceiving of the existence of such infinitely enduring motions is not greater than that inherent in other conceptions of which we frequently make use. For example certain properties of crystals, of unannealed glass, and of melted lavas are referred to conditions of stress which have persisted in these bodies for thousands of years, though one might think that the molecular motions constituting heat must have long ago equalised the stresses in question. Astronomical observation furnishes examples of bodies whose motions endure through ages.

Axial movements, such as we here suppose to take place in the field, have certain properties in relation to stability, as may be seen from the simple case of a spinning-top (law of free axes). Motions of this kind, in which the same succession of events occurs again and again—cyclical motions as they were appropriately termed by HELMHOLTZ—confer on the bodies which they actuate a certain stability which would not exist in the absence of such motions. Lord KELVIN has illustrated this by means of ingenious models, and has shown that a system of gyrostats may possess properties identical with those of an elastic solid.

'Permanent motions as apparent substances' was the title of an address which HELMHOLTZ was to have given at the Scientific Congress at Nuremberg in 1893, though unfortunately the congress never was held. Amongst his scientific remains only the introduction was discovered, though the author was fortunate enough to learn in conversation that the subject of the address was the persistent character of certain 'cyclic motions,' whereof vortex motion had already been investigated by HELMHOLTZ himself. See also under (*C*) and (*D*).

So far we have failed to find any means of satisfying ourselves directly of the existence of elementary rotations in magnetic fields; the motions in question belong to the class called 'hidden motions' (*verborgenen Bewegungen*) by HELMHOLTZ, and play a prominent part in the 'Prinzipien der Mechanik' of HEINRICH HERTZ.

In the second part of the book a complete exposition will be given of the highly suggestive kinetic theory of electro-magnetic and electro-dynamic phenomena, which in this work is only introduced by way of illustration.

B.—Mechanical interpretation of the dynamical properties of magnetic lines of force.

If a magnetic field behaves like a region filled with a rotatory motion of its minutest parts, we must attempt to discover how the dynamical properties of such a motion can account for the pressures and tensions in the field. To this end we must ascribe to the constituent motions a certain amount of energy—that is to say, we must suppose the thing which moves to possess inertia. This brings us to the consideration of MAXWELL's vortex theory. The assumption of Maxwellian molecular vortices has led to values for the density of the medium in motion which agree very closely with those deduced from other observations (GRÄTZ). We must here content ourselves with showing how the centrifugal tendency in a rotatory motion accounts qualitatively for the observed mechanical forces in the field, the rigorous investigation of MAXWELL (1861) being reserved for the second part of the book.

125. **Pressure and tension in a rotating element.**—That a rotatory motion gives rise to a tension and consequent tendency to shorten along the axis, accompanied by a pressure, or tendency to lateral expansion, in directions perpendicular to the axis, may be exemplified by means of a model, which we may take to represent an element of a tube of force (O. J. LODGE).

On the under face of the horizontal base-board C of a centrifugal apparatus there is a grooved pulley r which can turn freely about a vertical axis, and may be set in rapid rotation by means of the connecting band L, which couples it to a large driving wheel lying to the right, outside the range of the figure. On the axis of the rotating pulley an elastic cylindrical drum (shaded in the figure) is fastened by means of a thumbscrew F. This consists of two circular wooden discs D, D, above and below, over which the thick-walled rubber tube G is passed, and bound round with wire at s_1, s_2. In the upper disc are two holes closed by corks K, K, while to the lower one is attached the spindle A, which can turn freely in a hole drilled through the fixed support H. Weights may be hung from the lower end *e* of the axle A. Through the holes in the upper disc liquid is poured into the drum, mercury being used if the drum is very thick-walled, though otherwise, especially when the drum is of greater size (12 cm. high, 10 cm. in diameter), glycerine will be better.

If the cylindrical element G, filled with fluid (fig. 36), be set in rapid rotation about its axis, its elastic walls become bulged out laterally. At the same time the length of the drum will be diminished, as if its two ends attracted one another. The tendency to contract is so great that a considerable load may be attached to *e*, and will be lifted up. There is also a pressure exerted on the walls of the vessel, in directions perpendicular to the axis. If we imagine a large number of such rotating cylinders to be closely packed together, they will mutually hinder one another from undergoing a change of shape, and we shall have a mechanical representation of the tension along the lines of force and the pressure across them.

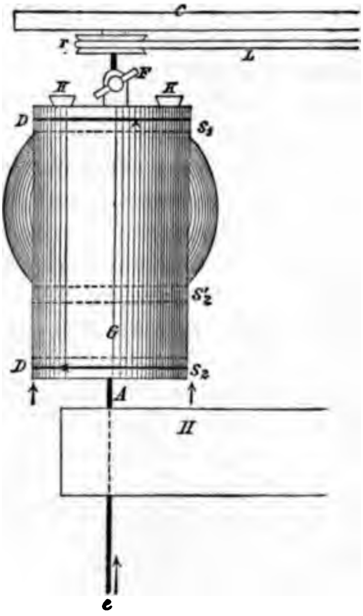

FIG. 36

This model furnishes an instructive example of the way in which a distribution of motion may give rise to apparent actions at a distance, the motions themselves being hidden from our observation. It might be called a gyrostatic model of a tube of force. The kinetic energy of its rotation is at each instant proportional to the angular velocity q (see also below, under *D*), and from § 119 we know that the energy per unit volume of a magnetic field is proportional to the square of $\mathfrak{H}$, so that we may consider the angular velocity q in our mechanical model as the analogue of the field intensity $\mathfrak{H}$.

126. Explanation of the mechanical forces due to magnets, in accordance with the hypothesis of permanent motions.—We must conceive of a magnetic field as being completely filled with rotatory motion. That such a distribution of motion is possible without the constituent motions disturbing one another we know from the phenomena of vortices (see also below, under *C*).

If two fields are superposed upon one another, the tensions along the lines of force, and the pressures across them, increase with the angular velocities. If the body giving rise to one field is fixed, while that giving rise to the other field is movable, the latter body will be displaced by the forces acting upon it.

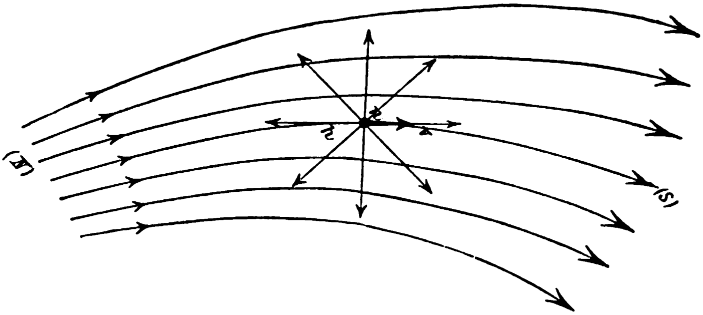

FIG. 37

(*a*) *Behaviour of a north pole in a magnetic field.*—Fig. 37 represents a section taken through any given fixed field of force, the positive direction along each line of force being indicated by the arrows attached to it. We must accordingly suppose the lines to be proceeding from a north pole to the left, towards a south pole to the right. Let a north pole n be placed in the field; it behaves as a source of new lines of force, which spread out from it in all directions indifferently. Eight of these lines are indicated in the figure, that marked *v* being coincident in direction with the fixed field. In the case of this line the magnetic effects are increased by the superposition of the two fields, so that there is a corresponding increase in the energy of the field and in the tension along the lines of force. In the case of the line *h*, the field due to the pole is opposed in direction to the original field, so that there is a corresponding diminution of energy, and decrease in the tension. Hence the pole will be urged in the direction of *v*—that is, in the positive direction of the lines of force.

A north magnetic pole, when placed in a magnetic field, experiences a force in the positive direction of the lines of force.

The effect is increased by the lines of force which pass out in oblique directions from the pole. All lines which are directed forward add something to the tension along the lines of force of the field, while with lines whose general direction is backwards, the opposite is the case (*cf.* fig. 30). In fig. 37 the direction in which the pole tends to move is indicated by a short thick arrow; if motion in that direction is prevented by some constraint, this latter experiences a thrust, similar to the thrust exerted by a mass placed in the earth's gravitational field and not allowed to fall.

(β) *Behaviour of a south pole in a magnetic field.*—If a south pole *s* (fig. 38) be introduced into the same fixed field, the effects observed will be exactly the opposite of the foregoing. For a south pole behaves as a sink towards the lines of force, lines converging towards it and there ending, instead of diverging from it. This is indicated in fig. 38 by the arrows on the eight lines of force which are directed towards the pole, and represent its field. As before we have to distinguish between a region in front of the pole and one behind it, considered in relation to the lines of force of the original field which run from *N* towards *S*. The lines proceeding from *h* towards the pole increase the intensity in their part of the field, giving rise to greater energy per unit volume, and correspondingly greater tension. In the case of *v*, the opposite takes place, with the result that the pole is urged backwards, towards *h*.

A south magnetic pole, when placed in a magnetic field, experiences a force against the positive direction of the lines of force.

In fig. 38 the direction in which the pole tends to move is indicated by a short thick arrow.

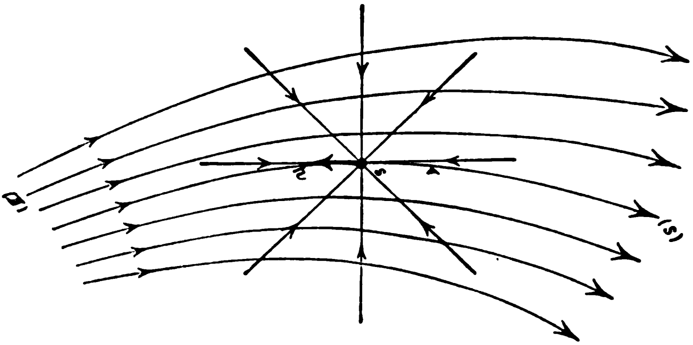

FIG. 38

127. **Kinematic representation of tubes of force.**—It might appear difficult, from the infinity of rotatory motions in a magnetic field, to obtain a clear mental picture of the process. If, however, we confine our attention to a single tube of force, all the particles at its bounding surface will be moving in the same direction of rotation. If any such particle, in the course of its rotation about its line of force, passes into the interior of the tube, its place on the surface is immediately filled by a particle of the same kind, moving in the same manner. The tube of force, as we conceive it, is a distribution of motion taking place always in the same sense, the kinematical conception being of a simple kind.

Suitable models of tubes of force may be made from pieces of white rubber tubing (fig. 39), a series of arrows being drawn round at equal intervals to represent the sense of the rotatory motion, and arrows along the length of the tube to indicate the direction of the magnetic field intensity. The two sets of arrows must have their directions related to one another in the manner determined by their convention of § 121. The tubes may be rendered less flexible by threading soft copper wires through them, and they may then be bent into any desired form. If we wish to make a tube returning into itself, the two ends may be kept

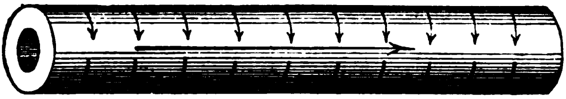

FIG. 39

together by fitting them over a short cylindrical piece of wood, of suitable diameter.

128. **Dynamical models of several important forms of magnets.**—By means of the model of a tube of force described in the last paragraph, we shall be able to picture to ourselves the rotatory motion assumed to exist in the field of a magnet.

(*a*) *Model of a bar-magnet*, fig. 40. The magnet is represented by a wooden bar N S, supported in a horizontal or vertical position upon a stand H, the end N (north-pointing end) being painted red, and the end S (south-pointing end), blue, while J represents the indifferent zone. At each end of the bar six holes are bored, equally spaced out from one another, and into these holes are stuck as many thick copper wires, enclosed by rubber

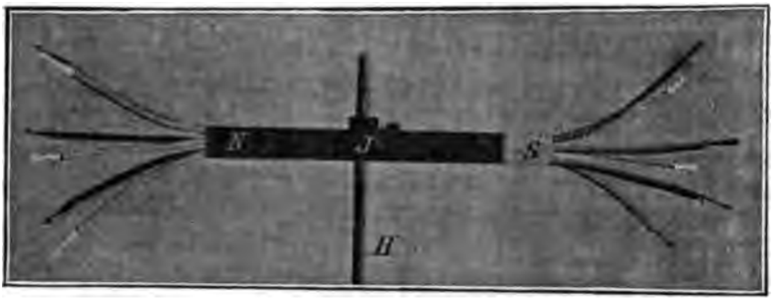

FIG. 40

tubes, which are marked with arrows to indicate the direction of rotation. To the ends of the wires, little tin arrows are soldered to show the corresponding direction of the magnetic force. The wires are then bent so as to follow the course of the tubes of force as they emerge from N and re-enter at S.

(*b*) *Model of a magnetic shell*, fig. 41. A wooden disc S, painted red on one side (*n*) and blue on the other (*s*) is attached to a brass support H, which stands upon a foot F. Through holes bored at equal intervals through the disc are stuck nineteen thick copper wires, 20 cm. in length; one wire through the centre of the disc, six in a circle surrounding this, and twelve more in a larger circle surrounding the former. Over these wires tubes are drawn as in the previous model, and the wings and heads of little tin arrows are soldered to their respective ends. The copper wires are curved out from one another in the manner shown in the figure, so that there is a bundle of them diverging from each side of the wooden disc

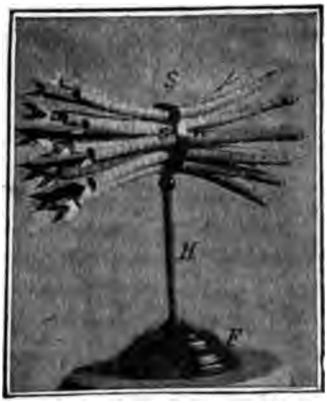

Fig. 41

(*c*) *Model of a magnet internally traversed by tubes of force*, fig. 42. The chains of molecules continue through the interior of the magnet the flux of force of the field, and this may be represented by means of the following model:

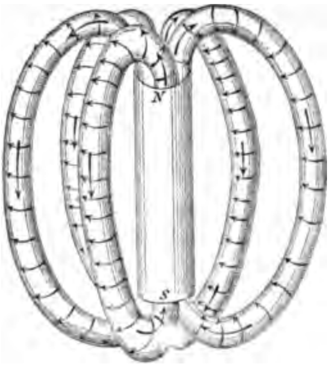

Fig. 42

A pasteboard tube 20 cm. long and $2\frac{1}{2}$ cm. in diameter has red paper pasted over one end, and blue paper over the other to represent their polarities. Five pieces of tubing, 60 cm. in length, are passed through this tube, and the ends of each piece are joined together, to represent a closed tube of force. The external portions of the pieces of tubing are curved so as to follow the form of the lines of force.

In reality, the bundle of tubes would become considerably constricted on passing into the interior of the magnet.

C.—Experiments on vortex motion

Kinetic or dynamical theories seek to explain the mutual actions of magnets by means of a distribution of motion in the surrounding medium. Rotatory motions about the magnetic lines of force are in reality capable of furnishing a mechanical explanation of the fundamental phenomena in question. The rotation in two neighbouring tubes being in the same sense, the particles at their peripheries, where they are in contact, will be moving in opposite directions, and it might appear as if the motions must mutually disturb one another.

We know, however, from the phenomena of vortex-motion in a frictionless fluid that such rotatory movements may be:

(*a*) *permanent*, and

(*b*) *in juxtaposition*.—Motions of this form, which were first accurately studied by HELMHOLTZ, may under certain conditions become permanent motions of the kind here considered. Since they constitute the best dynamical illustration of the kinetic theory of the field, we shall generate them in a fluid medium which can be made to fall in drops (water), and also in a compressible fluid (air). The resulting phenomena are indeed only approximations to those which are assumed in our theory of the magnetic field, for there are two conditions which must be fulfilled by our ideal medium, and which are not fulfilled by the substances used in our experiments. These conditions are:

(*α*) *Frictionlessness*, or freedom from viscosity.

(*β*) *Incompressibility*. Water possesses a certain degree of viscosity, so that the energy of the motion considered is gradually reduced to nothing; while air, in addition to having viscosity, is highly compressible. But the phenomena of vortex motion in these fluids may be made to furnish quite satisfactory analogues of lines of force, bundles of lines, and tubes of force

129. Generation of vortices in water.—Vortex motion is produced in a fluid (possessing some viscosity) when the fluid is made to flow round sharp edges.

Von Helmholtz first studied the vortices produced on stirring milk and coffee together in a cup, the different colouring of the two liquids rendering the motion easy to follow.

In our experiments we may use a glass trough (parlour aquarium) 3 to 4 decimetres in length, breadth and height; a little sawdust being added, which shares the motion of the water, and thus enables us to form an idea of the rotational velocity of each element of fluid, and of the direction in space of its axis of rotation.

Experiment.—A thick sharp-edged piece of sheet metal, whose lower boundary is semi-circular, is held perpendicularly to the free surface of the water, and is moved forward horizontally, parallel to itself, through about 8 or 10 cm., being lifted suddenly out of the water when it comes within 2 or 3 cm. of the side of the vessel. In this way a vortex is produced having the form of a half ring, terminating at the free surface of the water, and travelling forward towards the wall of the vessel. The edge of the metal plate moves along with it a filament of fluid in which the rotatory motion may be clearly seen.

130. Vortex motion as a permanent motion.—When a vortex exists in a frictionless fluid, the rotational motion is always associated with the same fluid particles, which move along with the vortex.

Thus in the experiment described above, particles of sawdust inside or outside the vortex-ring, and not originally actuated by rotational motion, did not acquire rotation, even when the ring travelled forward, opening out wider as it progressed.

Accordingly the rotational motion only affects a perfectly definite collection of fluid particles.

The vortex motion is thus permanently associated with a determinate set of fluid particles, and always travels along with them.

Herein lies an essential difference between vortex motion and wave motion, which travels along, spreading out more widely as it goes, being continually transmitted from one set of particles to another, while each separate particle executes an alternating movement about its mean position, from which it is never displaced very far.

Once vortex motion exists in a frictionless fluid, there are evidently two sorts of particles to be considered: particles having a rotational motion; these maintain their rotation so long as the conditions of motion remain undisturbed: particles having no rotational motion, and destined never to acquire it. The effect of fluid friction is to diminish the energy of the vortex motion, while at the same time causing it to spread to particles originally free from it, but in a truly frictionless fluid the two classes of particles would remain for ever distinct from one another, just as the molecules of a magnet emit permanent lines of force with which they appear to be unalterably associated. HELMHOLTZ has shown that in a fluid free from viscosity vortex motion once existing would remain for ever indestructible.

131. **Vortex lines in the vortex filaments.**—The individual particles of fluid in a vortex may be conceived of as lying along lines which run through the interior of the vortex from one end to the other.

If, for example, we fix our attention on a particle near the middle of our vortex ring, we see that its axis of rotation is parallel to the boundary of the entire vortex. On either side, neighbouring particles are turning about axes which appear to be prolongations of the first-mentioned axis. Thus we may pass in the direction of the axis from one particle to the next, from that to another, and so on, so that the path which we follow is at every point in the direction of the local axis of rotation.

In a fluid which is the seat of vortex motion, lines can be determined such that at every point the particles rotate about them as axes. These are the so-called '*vortex lines.*'

In our example these lines are semicircular in form. They originate at a point in the free surface of the liquid, and pass downward through the liquid, finally bending upward, and terminating at another place in the free surface.

In the dynamical illustration, the vortex lines are the analogues of the lines of force, while the bundles of neighbouring vortex lines correspond to the bundles of lines of force. Following HELMHOLTZ we shall apply the name 'vortex filament' to an indefinitely thin strip of fluid filled and entirely bounded by vortex lines.

In an ideal frictionless fluid a vortex line always passes through the same set of rotationally moving elements ; and a vortex filament, being made up of a collection of vortex lines, consists always of the same identical portions of fluid. It is the analogue of the tube of force, § 81.

The central vortex line may be called the axis of the filament ; the other vortex lines circulate continuously around it. The outermost lines constitute the wall of the tube, where the rotational motion is most apparent. The elements of fluid move round this wall in the sense corresponding to their rotational motion.

132. **Direction of the vortex lines.**—Like the lines of force, the vortex lines have a determinate direction, the positive sense along such a line being distinguished from the negative by the direction of rotation of the constituent elements of fluid. If we apply here the convention set forth in § 121, the semicircular vortex filament of fig. 43, whose

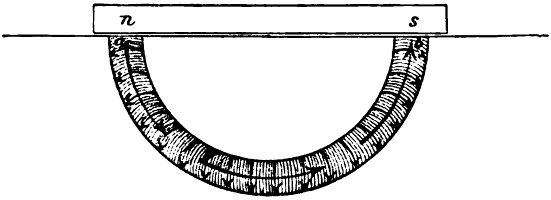

FIG. 43

extremities lie in the free surface of the fluid, will be taken as running in the direction of the arrow (positive sense along the filament).

At the point *a* we must conceive of a 'source' of vortex motion, the analogue of a north pole *n*; while at *b* there will be a corresponding 'sink,' or south pole *s*.

In the case of a vortex ring, there is another direction to be specified, namely that in which the ring as a whole moves forward; this is the same as the direction in which those particles are moving which have just reached the innermost portions of the bounding surface of the ring. Thus the vortex filament in fig. 43 would be moving towards the reader.

133. **Strength of a vortex.**—When a vortex filament travels through the fluid, widening out, and increasing in length, a change at the same time takes place in the velocity with which its constituent elements rotate about their axes. In any fluid, indeed, which we can examine experimentally, the entire motion would be ultimately annihilated by friction, but we may easily satisfy ourselves that a considerable increase takes place in the intensity of the rotational motion.

If, for example, vortex filaments are made to encounter the glass wall of the containing vessel, they spread out laterally at the moment of the encounter, and at the same time the rotational velocities of their constituent particles become greater.

When a vortex filament becomes extended to a greater length, the rotational velocity of its particles increases.

The product of the angular velocity $\mathfrak{q}$ and the cross-section ω of the vortex filament was called by Helmholtz the 'strength of the vortex,' $\mathfrak{J} = \mathfrak{q}\omega$. It is the analogue of the flux of induction $I = \mathfrak{B}\omega$.

134. **Conservation of the strength of a vortex.**—That the vortex strength $\mathfrak{J} = \mathfrak{q}\omega$ is a magnitude characteristic of each vortex filament is proved by the fact that it follows a law of conservation, like that which holds good for the flux of induction $I = \mathfrak{B}\omega$, or—in the special case where $\mu = 1$, so

that $\mathfrak{B}$ becomes identical with $\mathfrak{H}$—for the flux of force $\mathfrak{H}\omega$ (§ 83).

This extremely important law was established by Helmholtz, as a consequence of the theorem that the rotational velocity increases in proportion to the distance between two neighbouring particles of the same vortex line.

Since any given portion of a vortex filament always consists of identically the same liquid elements, and owing to the incompressibility of the liquid, the volume of this portion cannot change, the angular velocity $\mathfrak{q}$ must vary inversely as the cross-section; that is the product of the two must be constant.

Thus a vortex field is a complete mechanical analogue of a field of force.

As in the case of the latter the field may be supposed to be partitioned into tubes of force, along each of which the flux of force remains constant, so in the case of the former we have the constancy of vortex strength along each vortex filament.

The more rigorous mathematical analysis given in the second part of the book will enable us to follow the analogy further.

135. **Experiments with smoke rings.**—Smoke rings, such as a skilful smoker blows with his mouth, show very clearly the motion corresponding to a tube of force which runs completely round a closed curve, such as a circle. Tubes of force of this kind will have to be more particularly considered in Section II.

Smoke rings may be produced in greater numbers and with greater regularity by making use of a cylindrical tin or cardboard case placed with its axis horizontal, and having a circular hole pierced through the middle of its base. Smoke is blown in through the other end of the case, which is then closed by a lid. When the air within the case has come to rest, a succession of blows is struck upon the lid, and at each blow a well-formed ring issues from the circular hole in the base. The admixture of smoke particles enables us to follow and recognise the movements of the air expelled from the box, and it is always with this particular portion of air that the vortex motion is associated.

The rings which issue from the apparatus are best observed in the direction of their motion, and should be illuminated from in front or from behind.

In such smoke rings (fig. 44) we may observe the following peculiarities.

(*a*) Each ring is in the form of a closed vortex tube, its constituent filaments running round in closed curves, that is to say, in circles parallel to the circular axis of the ring, about which the separate smoke particles are seen to revolve.

(*b*) The vortex ring moves forward in the same direction in which the air passes through its aperture. This air is being continually drawn from the surrounding atmosphere, and is not charged with smoke. The smoke particles make it clear that it is the same portion of air which takes part in the vortex motion, from the commencement onward, and that none of this original air leaves the vortex, at least during the time for which the motion remains appreciable.

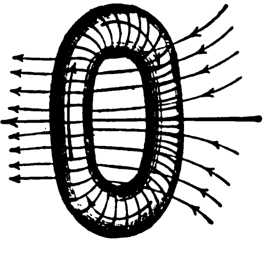

FIG. 44

(*c*) When we consider the smoke ring as a whole, the positive direction is to be determined in accordance with the convention of § 121, that an eye looking in the positive direction along the axis of rotation is to see the particles revolving round the axis in the direction of clock-hands. Since in each cross-section of the ring the motion at the inner part of the circumference is forward, that is, in the direction in which the whole ring progresses, and at the outer part of the circumference is in the contrary direction, that direction round the ring is accordingly to be taken as positive which is seen as a clockwise revolution by an observer looking at the ring in the direction in which it progresses. Compare the arrows in fig. 44.

(*d*) The form of a ring, especially after it has travelled some distance, is frequently disturbed by draughts of air, so that one portion of the ring becomes elongated; in this case a simultaneous rapid increase of rotational velocity is observed, though this is soon reduced again by the influence of friction. The phenomenon is an illustration of that constancy of 'strength' which would characterise a vortex in a frictionless fluid.

(*e*) If a smoke ring is allowed to impinge perpendicularly upon a wall, such as a window-pane, it does not recoil, but spreads out into a larger circle. This causes an increase in the length of the vortex, and a corresponding decrease in its cross-section, the rotational velocity at the same time becoming greater—noticeably so at the first moment. (Constancy of strength of a vortex $\mathfrak{J}=q\omega$.)

D.—Cyclic motions

In all modern field theories an important part is played by 'cyclical' motions, gyrostatic and vortical. These motions have been systematically investigated by Helmholtz, with especial reference to the problems of thermodynamics. Hertz has further developed the same conceptions, in his 'Prinzipien der Mechanik,' and has shown how they may be applied to the subject before us.

Just as it is found convenient, in treating of the laws of acoustical phenomena, to commence with some account of the type of motion fundamental to the subject, namely wave-motion; so here we shall find it well to explain in some measure the 'cyclic motions' which we shall have to consider. For the present we shall confine our attention to simple and familiar examples, which may serve to illustrate the nature of these motions and their principal properties. In the second part of the book we shall enter at greater length into their theory, as elaborated by Helmholtz, and further developed by H. Hertz and others.

136. **Conception of a cyclic motion.**—When a system of bodies is in motion, there is in general a progressive change in the distribution of the bodies in space, accompanied by a

change in the condition of the system. Besides this kind of progressive motion, however, there are cyclical or continuously recurrent motions, wherein each particle, on being displaced from its position, is immediately replaced by a similar particle moving in precisely the same manner, so that in spite of the motion the condition of the system does not change.

The uniform rotation of a homogeneous body about an axis of symmetry, and the flow of fluid in a canal of ring-shaped or other re-entrant figure, are examples of such motions (HELMHOLTZ).

Following HELMHOLTZ we shall apply the name 'cyclic motions' to motions of this kind, adopting the following as our definition of a cyclic motion, that is to say, of a 'purely cyclic motion' (compare § 142, below):

Definition.—A purely cyclic motion of a material system is such that at each place in the system, when one particle moves from its position, it is replaced after an indefinitely short time by another particle having identical properties, and moving in the same direction with the same velocity.

This definition, taken in its strictest sense, would imply that the motion takes place in an absolutely continuous medium. If this is not the case, if we are dealing, for example, with ordinary ponderable matter, built up of separate molecules, and moving in the manner specified in the definition, we can only regard the motion as purely cyclic in so far as we can ignore the heterogeneity of the matter.

Model of a simple cycle.—A wooden disc, about 10 cm. in radius and 2 cm. thick, is free to turn about a horizontal axis, which is supported on a vertical stand. At unit distance from the axis, a peg is inserted into the disc, to serve as a point of application for the force which is to set the disc in motion. Fastened to the stand, in front of the disc, and pointing towards the axis, is a second peg, which nearly, but not quite, touches the first as the disc goes round, and which serves to mark out a determinate initial direction. Then the position of the disc at any instant is given by the angular displacement of the first peg from the initial direction. If the disc is made to turn with constant velocity about its axis, each position, as it is vacated by one particle, will be immediately occupied by another, having the same properties and the same motion. If we leave out of account the minute heterogeneity of the material, we may say that the configuration of the whole system remains unchanged during the motion. If the rotation is made sufficiently rapid, it ceases to be evident to our observation, and we have in a certain sense an example of 'hidden motion,' of whose existence we may satisfy ourselves through the medium of the sense of touch.

137. **Cyclic co-ordinates and cyclic velocities.**—The positions occupied by the constituent parts of any movable system may be defined by fixing the magnitudes of certain quantities which vary with the time, and are called 'co-ordinates.'

This term, then, has here a more general meaning than in Cartesian co-ordinate geometry, where the rectangular co-ordinates of a point, for example, are its distances from three co-ordinate planes. A co-ordinate is any quantity (variable with the time) which helps to define the position of a body or of one of its parts, or in any way to determine the configuration of the system, whether the quantity by itself is sufficient to fix the configuration completely, or whether other co-ordinates must be known in addition.

In order to follow as closely as possible the notation used by Helmholtz and by Hertz, we shall use the symbol $\mathfrak{p}$ to denote co-ordinates of a system in cyclic motion, so that $\mathfrak{p}$ then represents a 'cyclic co-ordinate.'

In the example of the model in § 136 the cyclic co-ordinate $\mathfrak{p}$ will be most conveniently chosen as the ever-increasing angular displacement of the peg in the rotating disc from its original position.

If $d\mathfrak{p}$ is the change occurring in the value of $\mathfrak{p}$ during the infinitesimal time dt, then

$$d\mathfrak{p}/dt = \dot{\mathfrak{p}} = \mathfrak{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

is the rate of change of the cyclic co-ordinate, that is the cyclic velocity.

In our example, $\mathfrak{q}$ is the angular velocity with which the disc rotates.

The velocity of any point in the rotating disc distant r_i from the axis is $v_i = r_i\mathfrak{q}$. The velocities of all elements of mass taking part in the cyclic motion are thus expressible in terms of the single cyclic velocity $\mathfrak{q}$, to which they are directly proportional. The factor r_i depends only on invariable geometrical circumstances, namely, the distance of the point in question from the axis of rotation.

An essential property of the Helmholtzian cyclic system is this: that its condition does not depend on the instantaneous value of the cyclic co-ordinate $\mathfrak{p}$ itself, but only on its rate of variation with respect to time, that is, on the cyclic velocity $\mathfrak{q}$.

As a matter of fact, if every part of the system is exactly similar to every other part, and even the point of application of the applied force is not directly evident to our senses, there is nothing to tell us at any given moment what value $\mathfrak{p}$ has attained, since like particles continually follow like. Thus the absolute value of $\mathfrak{p}$ does not concern us; when the cyclic velocity $\mathfrak{q}$ is known, everything relating to the system is determinate.

In our example of a uniform circular disc, rotating with constant velocity $\mathfrak{q}$, this is immediately evident.

138. **Energy of cyclic systems; the cyclic moment.**—Since in a cyclic system the velocity of each constituent part is determined by a single cyclic velocity, the same will hold good for the energy of each element of mass, and consequently for the energy of the whole system.

This may be immediately seen in the case of a body rotating about an axis, for example, our circular disc, § 136. A particle of mass m whose distance from the axis is r_i will be moving with the velocity $v_i = r_i\mathfrak{q}$, and its kinetic energy will be $\frac{1}{2} m_i r_i^2 \mathfrak{q}^2$; the kinetic energy of the whole system being thus

$$\mathfrak{E} = \tfrac{1}{2}\Sigma(m_i r_i^2)\mathfrak{q}^2,$$

or when we put

$$\Sigma m_i r_i^2 = \mathrm{T},$$

where T is the moment of inertia of the disc

$$\mathfrak{E} = \tfrac{1}{2}\mathrm{T}\mathfrak{q}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Thus in the case of rotation about a fixed axis, the expression for the kinetic energy involves the moment of inertia in place of the mass.

The differential coefficient of the kinetic energy with respect to the velocity $\mathfrak{q}$ is called the cyclic moment

$$\mathfrak{f}=\frac{d\mathfrak{L}}{d\mathfrak{q}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (21)$$

also, $\mathfrak{L}=\frac{1}{2}\mathfrak{q}\mathfrak{f}$.

In our example, $\mathfrak{f}=\mathrm{T}\mathfrak{q}$, so that $\mathfrak{L}=\frac{1}{2}\mathfrak{q}\mathfrak{f}$. Again $d\mathfrak{f}=\mathrm{T}d\mathfrak{q}$ (T constant), and the energy differential has the form

$$d\mathfrak{L}=\mathrm{T}\mathfrak{q}.d\mathfrak{q}=\mathfrak{q}d\mathfrak{f} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (22)$$

This expression like the differential expressions in § 118 and in § 119 (16) is the product of two factors.

139. Forces corresponding to cyclic co-ordinates.—Let the motion in the cycle be frictionless, and determined by the cyclic velocity $\mathfrak{q}$; the system will possess a certain amount of kinetic energy. If $\mathfrak{q}$ is increased to $\mathfrak{q}+d\mathfrak{q}$ by allowing a force $\mathfrak{P}$ to act in a definite manner, a certain amount of work d will be performed, and since nothing in the system is changed except $\mathfrak{q}$, the work $d\mathfrak{U}$ done on the system must be equal to the increase $d\mathfrak{L}$ in the kinetic energy.

In our example

$$d\mathfrak{U}=d\mathfrak{L}=\mathrm{T}\mathfrak{q}d\mathfrak{q}=\mathfrak{q}d\mathfrak{f}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23)$$

We define the (generalised) force ('Kraftmomente,' HELMHOLTZ) corresponding to any one of the variables by supposing this variable alone to change by an infinitesimal amount and dividing the infinitesimal work done by the infinitesimal change in the variable. In our case the work $d\mathfrak{U}$ is performed on the point of application (the peg inserted in the disc) whose position is determined by the co-ordinate $\mathfrak{p}$. We have thus for the generalised force (in this case the *moment* of the applied force about the axis of rotation)

$$\mathfrak{P}=\frac{d\mathfrak{U}}{d\mathfrak{p}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (24)$$

and

$$d\mathfrak{U}=\mathfrak{P}d\mathfrak{p}=\mathfrak{P}\mathfrak{q}dt \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (25)$$

This definition of the force is more general than that in ordinary mechanics; it includes not only forces, and moments of forces, but also applies to cases in which the condition of a substance alters, and where we do not ordinarily speak of forces at all; for example there is the pressure of a gas, which may be deduced from the work required to produce an infinitesimal decrease of volume. A comparison of (25) and (23) gives for our case

$$\mathfrak{P}\mathfrak{q}dt = T\mathfrak{q}d\mathfrak{q} = \mathfrak{q}d\mathfrak{s}$$

or

$$\mathfrak{P} = \frac{d\mathfrak{s}}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The time-rate of change of the cyclic moment is equal to the (generalised) force acting upon the corresponding cyclic co-ordinate.

140. **Extension of the conception of a cycle.**—The *type* of the cyclic motion remains unaltered, even when in course of time a change takes place in the cyclic velocity $\mathfrak{q}$. We may consider a system to have at each instant a purely cyclic motion even when the cyclic velocity $\mathfrak{q}$ is changing, provided that the rate of change $d\mathfrak{q}/dt = \mathfrak{q}$ is sufficiently small. In this case the representation of the system (configuration of all elements of mass, their velocity and kinetic energy, and the energy of the entire system) involves the following quantities:

Co-ordinate $\mathfrak{p}$: does not enter into the dynamical expressions.

Velocity $\mathfrak{q}$ (rate of change of $\mathfrak{p}$): determines the condition of the system.

Acceleration $\frac{d\mathfrak{q}}{dt}$: is small.

141. **Coupled cycles.**—So far we have always had in mind a very simple case of cyclic motion. But there are also very complex systems whose motion is cyclic in character, so that the conception, as well as the formulæ given above, have a very wide range of application. If we

have two discs, of different diameters and of different materials, capable of turning about two parallel axes, and if we connect them by an endless band, the definition of § 136 is strictly applicable to this system, whose motion constitutes a pure, simple cycle; for each particle as it moves forward is replaced by another, similar in properties and moving in the same manner, so that in spite of the motion no change takes place in the configuration of the system. Even in the connecting band one part as it moves along is replaced by another, and if the band is quite uniform throughout its length, there is no change in its entire configuration, externally considered.

As cyclic co-ordinate it will be simplest to take the angular displacement $\mathfrak{p}$ of a definite point P of one disc from the position which it initially occupied. Since there is nothing to distinguish one particle from another, the absolute value of the co-ordinate $\mathfrak{p}$ has no dynamical significance. But its differential coefficient $\mathfrak{q}$ with respect to the time determines the velocities of all the particles, and hence their kinetic energy, as well as the energy of the whole system.

The gearing together of two or more circular motions in this way is called ‘coupling.’

Model of coupled cycles.—To a baseboard two axles are fastened. About one of these turns a simple wooden disc (a) of radius R_1 with a groove cut in its rim; about the other turns a double disc, consisting of a smaller disc of radius r_2 with a grooved rim, and a large disc of radius R_2 rigidly connected with it. The circumferences of R_1 and r_2 are connected by means of an endless rubber band vv. The dotted line a denotes the arbitrarily chosen initial position of a point P (not shown) which is fixed with respect to the disc, and is at unit distance from its axis of rotation. Its angular displacement $\mathfrak{p}$ from the position a is the cyclic co-ordinate of the whole system, and the force $\mathfrak{P}$

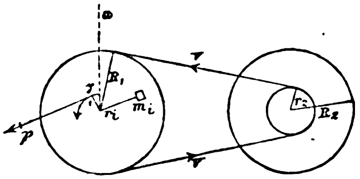

FIG. 45

must be conceived of as acting at P in the direction of the tangent to its path. [$\mathfrak{P}$ is only a force in the generalised sense; when $\mathfrak{p}$ is an angular magnitude, as in the present case, $\mathfrak{P}$ is the *moment* of the impressed forces about the axis of rotation. This being understood, the point P need not be at unit distance from the axis.] For an elementary mass m_i of the first disc, at the distance r_i from the axis, the velocity is $v_i = r_i \mathfrak{q}$, and the kinetic energy $\frac{1}{2} m_i r_i^2 \mathfrak{q}^2$; for an element of mass m_i' of the second disc at the distance r_i' from the second axis of rotation these quantities become respectively $R_1/r_2 . r_i' \mathfrak{q}$, $\frac{1}{2} m_i' (R_1/r_2)^2 r_i'^2 \mathfrak{q}^2$. The speed-ratio R_1/r_2 is a constant characteristic of the particular piece of mechanism.

In the cycle now under consideration the velocity at any point i takes the form $v_i = a_i \mathfrak{q}$, where a_i is a factor depending on the manner in which the points i and P are connected with one another. For all points on the first disc, a_i is numerically equal to the distance from the axis of rotation; for points in the connecting band, whose motion is in part rectilinear, in part circular, the value of a_i is simply R_1, the radius of the first disc; for all points in the second disc a_i is equal to the distance from the axis of rotation, multiplied by the speed-ratio R_1/r_2.

All arrangements of homogeneous discs connected by bands, and quite generally all combinations of homogeneous bodies turning about axes of symmetry connected by bands or strings (without slipping), give rise to pure cyclic motions.

Thus a cyclic motion is not necessarily a simple rotation about an axis; cyclic motions about axes any distance apart may be so combined as to constitute a single cycle. It is not even necessary that all the motions concerned in the cycle should take place along *circular* paths, some of them may be in part rectilinear, or (when the connecting bands are slack) may take place along catenaries. The fundamental character of the motion is still maintained, each element of the band, as it moves along, being always replaced by an identically similar element.

Again, substances of widely different kinds may take part in the cyclic motion: the substances of the discs and of the connecting band. Finally the axes about which the different motions take place need not be parallel to one another, and the directions of some of the rotations may be reversed by putting the bands on cross-wise, but still the fundamental character of the cyclic motion is maintained.

We have thus reached a considerable extension of the conception of cyclic motions. We may conceive of a region (field), filled with an immense number of cyclic motions, gyrostatic or vortical, all of which taken together constitute a single cyclic motion. The condition for this is that all the separate motions are determinable from a single cyclic velocity.

In the second section we shall become acquainted with such a system of coupled cyclic motions. It is not to be supposed that in nature the cycles concerned are coupled together in the simple manner of our discs with a connecting band, or that they have the same simple character. Our model only serves as an illustration of the principles, and helps to explain the manner in which the energy of the field is distributed.

When the velocities of the separate particles which take part in such a cyclic motion are proportional to the cyclic velocity $\mathfrak{q}$, the formulæ 19 to 22 hold good which we obtained in a very simple case.

For example, the total kinetic energy of the coupled cycles in fig. 45 is

$$\mathfrak{L}=\tfrac{1}{2}\left\{\ \Sigma_1 m_i r_i^2+\left(\frac{R_1}{r_2}\right)^2\Sigma_2 m_i' r_i'^2\right)\mathfrak{q}^2$$

where the first summation is extended to all particles of the first disc, and the second summation similarly refers to the double disc. The bracketed expression may be regarded as the moment of inertia T corresponding to the whole system. From $\mathfrak{L}=\frac{1}{2}T\mathfrak{q}^2$, however, all other formulæ follow, including equation (26).

If we compare the formulæ 20 and 22 for the energy of a cyclic motion with the formulæ 17 and 16, § 119, for the energy of a field, we see that the analogy is complete, a result which adds greatly to the plausibility of our attempt to explain magnetic phenomena by means of cyclic motions. If we do not wish to conceive of the energy of the magnetic field as being of some definite mechanical form (such as the kinetic energy in the present illustration) we must look upon the cyclic motions as constituting only instructive energy models, showing how we may imagine a realisable distribution of energy in the field.

142. **Imperfect cycles.**—The following example will serve to illustrate how the conception of a cycle may be extended in yet another direction. A wheel with spokes is made to rotate about its axle with constant angular velocity. Can the system of moving mass-elements be considered to constitute a cycle, as defined in § 136? In the rim of the wheel and in the hub, similar particles having similar motion succeed one another without intermission, so that to this extent the system is purely cyclic. But at any given place in the track of the spokes it is only during a limited time that particles of like nature (such as those of the spokes) continuously succeed one another; after the lapse of this time other particles appear in their stead, in the present case the irregularly moving particles of air. At each such place of the region in question this process is repeated periodically, so long as the character of the motion is maintained.

The configuration of the whole system, in regard to the point considered, is identically repeated after the lapse of each period. Here again the velocity of any particle i of mass m_i is $v_i = r_i . \mathfrak{q}$, where r_i is the distance of i from the axis of rotation and $\mathfrak{q}$ denotes the angular velocity of the wheel. Thus the kinetic energies of the separate particles, and their moments of inertia, as well as the kinetic energy and moment of inertia of the entire system, depend only on the cyclic velocity, just as in the case of a pure cycle. The wheel with spokes is an example of an imperfect cycle. The same holds good for a toothed wheel, or for a system of toothed wheels geared together. In all such cases $v_i = a_i \mathfrak{q}$, where a_i is a factor depending on the nature of the mechanism.

Even a non-homogeneous body, which turns with constant velocity about any given fixed axis, constitutes an imperfect cycle, provided that no change takes place within the body during the rotation. For after the lapse of the period of rotation the same state of things is always exactly reproduced; and through a large number of such periods the character of the motion is maintained unaltered. In the case of imperfect as of perfect cycles, we may make the assumption that the cyclic velocity is accurately constant; or if any change takes place in this magnitude, it must be a very slow change.

143. **Monocycles, dicycles and polycycles.**—So far we have confined our attention to cyclic systems in which the velocities and energies of all mass-elements are determined by the rate of variation $\mathfrak{q}$ of a single co-ordinate $\mathfrak{p}$. Such systems are called by Helmholtz 'monocycles.' They are characterised by the relation $v_i = a_i\mathfrak{q}$.

Bodies of any shape rotating about fixed axes, and any combinations of such bodies, coupled or geared together, may be regarded as monocyclic systems, provided the progressive changes taking place in their configuration are slow compared with the cyclic motion proper.

In Section II., when we come to deal with the electric current, we shall find fields whose properties present the closest analogy to the mechanism of a monocyclic motion. In dealing with 'induction' we shall have to recur to equations (20)—(26), especially the last.

But in addition to monocycles there are other cyclic systems in which the velocity of each mass-element is determined by the values of *two independent* cyclic velocities $\mathfrak{q}_1$ and $\mathfrak{q}_2$, so that in the simplest case

$$v_i = a_i\mathfrak{q}_1 + b_i\mathfrak{q}_2$$

Such a system is called by Helmholtz a 'dicycle.'

If the velocities of the separate masses of a system involve the simultaneous values of more than two independent cyclic velocities,

$$v_i = a_i\mathfrak{q}_1 + b_i\mathfrak{q}_2 + c_i\mathfrak{q}_3 + \ . \ . \ .$$

the arrangement is called a 'polycyclic system' or 'polycycle.'

144. **Slowly varying parameters.**—The cyclic motions occurring in nature are commonly of such a kind that they are repeated many times over before any considerable change takes place in the character of the motion. But a gradual change may be going on all the time, so that the later cycles are accomplished under conditions different from those of the earlier cycles. These gradual progressive changes we may suppose to be slow in comparison with the more rapid cyclic motion itself; the value of the cyclic velocity determines whether the change in question is to be considered slow or rapid.

Thus the rotation of the earth about its axis is a cyclic motion completed in a day. After the lapse of many days and years, precession and nutation will have produced a small change in the direction of the earth's axis; the change being 'slow' in comparison with the diurnal cyclic motion.

Quantities which determine the condition of a body or of a system of bodies are called 'parameters.' The parameter whose variations correspond to the slow progressive changes in the condition of the system we may refer to as 'slowly varying parameters.' If we denote them by the italic letter p, in distinction to the cyclic co-ordinate, these variables will be characterised by the following properties (compare § 140) :

Parameter p : appears in the dynamical equations.
q = rate of change of p : is small in comparison with q.
$\frac{dq}{dt}$ = rate of change of q : is very small.

SECTION II.

PHENOMENA OF THE GALVANIC CURRENT AND OF ELECTRO-MAGNETISM

Following the inductive method which is common to all the natural sciences, we shall commence this section with a study of the *phenomena* of 'galvanic currents,' without for the present concerning ourselves with their ultimate causes. In the same way the study of gravitational attractions has been highly developed and elaborated, although so far nothing like a satisfactory explanation has been given of the nature of the phenomena. We consider the current as a given natural phenomenon, and prefer the term 'galvanic' to the equally usual term 'electric' current, because in the study which we are about to make there is nothing to remind us of the effects produced by rubbed amber (ἤλεκτρον). Departing somewhat from the usual procedure, we shall commence with the magnetic properties of the current. These are the simplest, for they are conditioned only by the form of the conductor in which the current is flowing, and the nature of the surrounding medium. The properties of the material composing the conductor, and the processes which take place amongst its smallest particles, need not be taken in account.

We commence accordingly with the fields produced by currents, and their relation to the fields of magnets, that is, with the phenomena of electro-magnetism; the so-called current-strength being for the present regarded as merely a quantity determining the condition of the field. In the second part of the book, in connection with the phenomena of induction, we shall become acquainted with the most important methods of producing the galvanic current by means of magnetic action. Here we consider the current as a purely magnetic phenomenon, produced in the surrounding medium, especially by metallic bodies extended in the form of wires, when their ends are joined to so-called 'terminals.'

With what takes place beyond these terminals we are not at present concerned.

CHAPTER VIII

UNIAXAL MAGNETIC FIELDS OF FORCE (SIMPLE CURRENT)

In the present chapter we shall commence with an examination of the magnetic properties of a current flowing in a wire, investigating the structure of the surrounding field by following out the course of its lines of force.

To commence with the simplest possible case, we shall suppose the wire under investigation to be stretched along a straight line, all portions which are not straight, as well as the terminals to which the wire is connected, being so far off that no influence from them is to be feared. We shall thus obtain a very peculiar form of magnetic field, very different from those investigated in Section I. If, however, we bend the conductor round into the form of a loop, of which no two parts are allowed to touch, although approaching one another very closely, we have an arrangement whose properties are in many respects identical with those of a magnetic shell (§ 23).

It will be remembered that in § 120 we discussed the axial character of the magnetic lines of force and the kind of symmetry which they possess, and thus we arrive at other modes of representation which enable us, in our system of conceptions, to make use of the usual terms, 'current,' 'direction of a current,' 'conductor,' 'current path,' &c., without assuming, however, that there is something actually flowing along the conductor.

A special section will afterwards be devoted to explaining the essential differences between the fields of force due to magnets, and the field surrounding a current; it will be remarked that this difference is conditioned essentially by the distribution in space of the two forms of field.

A.—Coaxal systems of lines of force

145. The galvanic current as a system of magnetic lines of force.—We must now show that in the neighbourhood of a galvanic current magnetic effects are produced, proceeding next to the closer investigation of the corresponding system of lines of force.

Experiment 51.—A copper wire, about 5 mm. thick and 60 cm. long, somewhat bent round at the ends, is laid in a shallow wooden trough lined with white paper, and is uniformly sprinkled over with iron filings. No particles remain hanging to the wire, except a few from the effects of ordinary cohesion. If, however, by means of flexible conductors and brass binding screws the copper wire is joined to metallic terminals, a thick layer of iron particles remains in close contact with the wire, enclosing it like a jacket. *The wire has become a source of magnetic influence.* When the circuit of the current is broken, the particles fall away from the wire.

We have here, then, a temporary magnet, in which no residual magnetism can be detected (*cf.* § 101) ; so that in this respect it differs essentially from the permanent magnets considered in Chapter I.

The wire will be evenly surrounded by ribs or bands of iron filings adhering end to end, no portion of its surface being especially conspicuous, as the polar regions of a magnet are ; the filings do not form tufts or strings, but closed rings surrounding the wire.

By using the methods of projection this important fundamental experiment may be shown to a large audience. A copper wire, which should not be too thin, is bent over a glass disc which is placed in the vertical part of the beam of a projection-lantern, the arrangement of two reflecting prisms mentioned in § 6 being adopted. On the disc iron filings are scattered. When the wire is joined to the terminals, the particles of iron rush to the wire and embrace it, and when the circuit is once more broken, they fall away from the wire again.

Experiment 51 naturally leads us to suppose that the magnetic field due to the current is in some special way dependent upon the direction of the conducting wire. As in the case of bar magnets (§ 21), we shall examine the constitution of the field in two planes at right angles to one another, thus obtaining a complete knowledge of the distribution of the lines of force in space.

(*a*) *Course of the lines of force in a plane parallel to the direction of the current, and near to the conducting wire.*—Let the straight wire which is to carry a current be fixed horizontally, a thin sheet of paper being laid over it.

The following arrangement, which we shall also employ in other experiments, is especially suitable for lecture demonstration:

In a board about 12 cm. broad and 20 cm. long a groove is cut parallel to the longer side, and of such a depth that a copper wire about 5 mm. in diameter can be completely sunk in it so as to lie flush with the upper face of the board. A sheet of thin white writing-paper is laid over all, and fastened down with drawing-pins.

When the paper is strewed with iron filings and gently tapped, the appearance presented is like that shown in fig. 46.

It will be seen that just above the wire the filings become aggregated together into fine ribs, running perpendicularly to the length of the wire. It is evident that in this case the force which causes the ordered disposition of the filings is strongest in the immediate neighbourhood of the wire; for on either side of the narrow ribbed patch there is a lighter strip from which the filings have been drawn away. At greater distances from the wire no special influence can be inferred from the arrangement of the filings, the fact being that in such

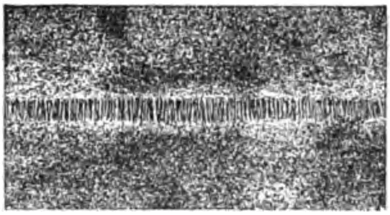

FIG. 46

places there is only a small component of the magnetic force lying in the plane of the paper.

(*b*) *Course of the lines of force in a plane perpendicular to the direction of the current.*—From the last experiment it is clearly evident that the lines of force run perpendicularly to the wire in its immediate neighbourhood; but we shall obtain a clearer insight into the distribution of the field when we investigate the line-of-force diagram in a horizontal plane, perpendicular to the (vertical) rectilinear wire.

To this end we may make use of the following simple arrangement: a wooden block 12 cm. × 15 cm. horizontally and 6 cm. thick has a hole 4 mm. in diameter bored through the middle of it, perpendicularly to its upper face, and through this hole passes a copper wire 4 mm. thick, the upper part of the wire projecting vertically above the hole, while the lower part is bent sideways, being buried in a groove which is cut in the lower face of the block. Upon the block is laid a sheet of paper, in the middle of which a circular hole has been punched, just large enough for the wire to pass through it. Terminals are connected to the two ends of the conducting wire.

If the experiment is to be shown to a larger audience, it will be better to use the following apparatus (fig. 47), which will be especially useful in some of our later investigations:

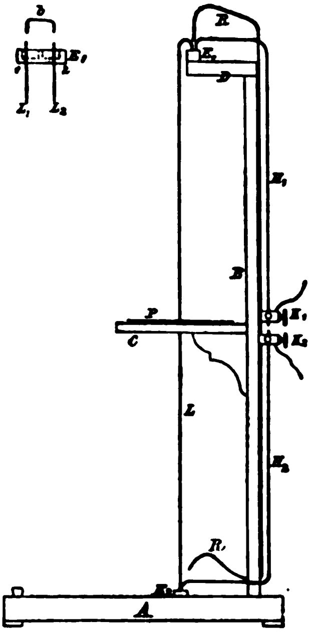

Fig. 47

Upon a base-board A, 40 cm. × 40 cm., is fastened the board B, 100 cm. high and 10 cm. broad. The base-board A stands on three feet, and has a moulding running round it, so as to form a kind of shallow trough. At one corner is an orifice which can be closed by a wooden plug (shown to the left in the figure), and facilitates the removal of any spilt mercury or scattered iron filings. At the middle of its height the stand B carries the small horizontal board C, 80 cm. × 20 cm. in extent. Through the middle of C a number of holes are bored, through which wires have to be passed; and upon it a sheet of paper P is laid. At the upper end of the stand B the shelf D, 10 cm. in length, is fastened. Upon D, and upon the base-board A, the little wooden blocks E_1 and E_2 are placed, each of which contains two equally deep hollows, separated from one another by a thin wooden partition, and filled with mercury. (Compare the small figure to the left, which shows a section of one of these wooden blocks, as seen from in front.) The middle line of the troughs in E_1 is about 8 cm. from the front face of B, the corresponding distance for E_2 being about 10 cm. To each of the wooden blocks leads a conducting wire H_1 from the terminal K_1 or H_2 from K_2 as the case may be. In the experiments now to be described, only a straight conductor L will be used. This is made of copper wire 4 mm. thick, and at the upper end is bent into the form of a hook. Its two ends are amalgamated, one dipping into a compartment of the block E_1 and the other into a compartment of E_2, the wire itself passing through a corresponding hole in the board C and paper P. In Chapter XII. we shall investigate the field due to the simultaneous existence of currents in two conducting wires, a second similarly shaped wire, L_2, being then placed beside the first. To enable us to join the two wires in circuit so that the current shall traverse them in the same sense, or in opposite senses as desired, there is an additional wire at the back of the stand, between the conductors H_1 and H_2, its ends R,R being capable of immersion in the mercury-cups; so that the upper end may be made to dip into either compartment of the upper block, and the lower end into either compartment of the lower block. A copper-wire link (*b* in the figure to the left) is used to connect two neighbouring mercury compartments, its ends being amalgamated. We shall have to speak later of the lines of force due to the simultaneous flow of two currents in neighbouring conductors.

All the wires H_1,H_2, and R,R are best made of a number of fine wires twisted together, and overwound with an insulating layer of silk or cotton; thicker pieces of wire are soldered to their ends. Such wire cords, even when quite thick, may be easily bent into any desired form.

Fig. 48 shows the line-of-force diagram obtained in a plane perpendicular to the rectilinear conductor. In the middle of the figure is seen the circular section S of the cylindrical wire, while the iron filings have arranged themselves in complete rings, embracing it concentrically. There are no points at which lines of force originate or terminate; in other words there are no sources or sinks in our field—no definite poles from which the magnetic influences proceed. All the lines of force form closed curves in the free field, thus differing from the lines which proceed from a permanent magnet after having traversed its interior along those chains of molecules whose existence we assume, but which always remain hypothetical.

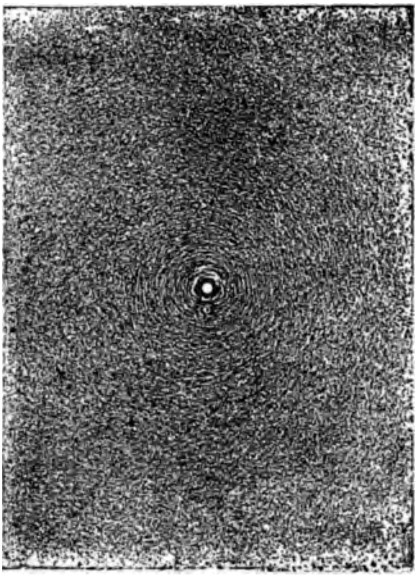

FIG. 48

The force must be greatest nearest to the wire, for here it will be observed that the filings are heaped together, leaving a surrounding patch nearly bare, a result which was always observed in our experiments on permanent magnets in places where the magnet effect was greatest. The filings to a distance of about 1 cm. from the wire join one another end to end; at distances of from 1 cm. to 2 cm. the aggregation is less close, while at still greater distances the magnetic force is insufficient to exert any appreciable directing influence on the filings. Those portions of the current conductor which are more than 4 cm. apart may thus be considered to have no sensible disturbing influence on one another's fields.

146. **Axes of concentric systems of magnetic lines of force or axes of force (Faraday).**—In every plane perpendicular to the rectilinear wire the lines of force are in the form of concentric circles, whose common centre is on the axis of the wire. Thus all the lines of force due to a current in a straight wire lie upon cylindrical surfaces whose axis coincides with that of the wire. This axis is called the axis of the field of force, the field itself being spoken of as axial. Following Faraday we shall use the shorter term 'axis of force,' which is not to be taken as implying any property of the conductor, except that it is the axis of a system of coaxal lines of force.

147. **Positive sense along the concentric circular lines of force.**—In order to determine whether there is a positive sense and a negative sense corresponding to the circular lines of force, we may use the freely turning bar magnet mentioned in Chapter II., or a short magnetic needle; later on we shall have to speak more particularly of the action between the fields of permanent magnets and fields of the kind now considered. For the present it will be sufficient to determine from the actions observed the sense in which the lines of force are to be reckoned positively.

Experiment 52.—A long thick copper wire is bent into the form of a rectangle, whose sides are about 50 cm. in length, and which is supported in a vertical position. A bar magnet of steel wire, about 1 metre long (§ 28), is arranged so that its lower pole swings freely at a distance of 1 or 2 cm. from a vertical side of the rectangle, in the neighbourhood of the middle point of that side. The other pole is then beyond the range of appreciable magnetic influence arising from the current in the wire. Moreover, the remaining portions of the rectangle are so far removed from the straight length of conductor under consideration that no appreciable disturbance from them is to be feared. When the extremities of the wire forming the rectangle are connected to 'terminals,' the pole of the suspended magnet is displaced to one side—that is, in a determinate direction along the lines of force of the axial system.

The pole cannot follow the lines of force right round the wire since the thin bar magnet is stopped by coming into contact with the upper side of the rectangle; nevertheless we are enabled to see quite clearly which is the positive sense along the lines.

If the positions of the poles be interchanged, the direction of the observed motion is reversed.

The axial magnetic lines of force have thus a determinate sense; a north pole being urged along them in this sense, and a south pole in the contrary sense.

This holds good for every portion of the wire and at all distances from it, throughout the range where the magnetic influences are appreciable. The entire axial system of lines of force has a determinate sense.

We can thus specify the positive direction of the current along the conductor in terms of the lines of force alone, so defining it that, to an eye looking along the conductor in the direction of the current, the lines of force would appear to embrace it in the clockwise direction. [Thus a translation in the sense of the current, combined with a rotation in the sense of the lines of force, would constitute a right-handed screw motion.] The considerations now to be adduced show that this definition accords with the usual conventions, while its convenience will be evident from the subsequent development of our system of conceptions.

If we interchange the connections between the terminals and the ends of the rectangularly bent wire, the reversal of the direction in which the current flows through the conductor is found to reverse the directions of all the lines of force in the system. This may be expressed by saying: the terminals, by whose means the magnetic effects are produced, have a polar contrariety of properties.

This polarity is ordinarily expressed by attaching to the terminals the signs + and —, or by means of different letters, such as K and Z, though these are not so expressive.

When a wire is arranged so as to connect a pair of terminals, we shall always consider its direction to be from + to — or from K to Z (these letters being consequently taken in alphabetical order). The results of all our experiments may then be stated as follows:

If we look along the conductor in the positive direction, a north pole will always be urged clockwise, and a south pole counter-clockwise along the lines of force of the coaxal system.

The lines of force are thus seen to pass clockwise around the axis of force by an eye looking along it in the sense from + to — or from K to Z.

We shall in future indicate the direction of the current in the wire by means of tin arrows painted red, or by pointing with the middle finger, just as white arrows and the forefinger were used to denote the direction of the magnetic lines of force.

Since the lines of force now considered also exert contrary actions on north and south poles, it follows that a short magnetic needle would tend to set itself in a direction perpendicular to the axis of force.

Experiment 53.—On the shelf C of the apparatus shown in fig. 47 is placed a small magnetic needle with pencil attachment (fig. 7), close to the wire and in the same magnetic meridian. When the ends of the wires are connected to the terminals, the needle turns round and sets itself perpendicularly to the meridian, *i.e.* to the plane passing through the wire and the pivot of the needle. If we look along the wire from + to —, and so that in our field of view the needle appears above the wire, then will the north pole be to the right hand and the south pole to the left. If the needle be moved in the direction of its length (reckoned from south pole to north pole) it will continually reset itself, so as to be always perpendicular to the line which joins its centre to the nearest part of the wire. Following out the successive positions of the needle, and continuing to move it always in the direction of its length, we shall have made a complete circuit around the wire, the cyclic direction being the same throughout. At each point of the line of force the needle indicates the direction of the tangent, and the curve traced on the paper by the pencil-point is very nearly a circle—not exactly, because the form is slightly disturbed by the earth's magnetic field.

If the connections of the ends of the wire with the terminals are interchanged, the needle turns round end for end, the entire system of lines of force being reversed in direction.

Henceforward we shall make use of one or other of the numerous devices which serve to reverse the direction of a current in a conductor. The unfastening and interchanging of the connections with the terminals is inconvenient, slow, and—owing to the production of sparks—somewhat unpleasant. It is better to use a combination of conducting pieces which enable us to effect a rapid reversal by depressing a switch or rotating a drum. Such an apparatus is called a commutator or reversing key, numerous forms being in common use. We shall use in connection with it a red arrow to indicate in each case the direction from + to − in the conductor.

148. **Motions in concentric systems of lines of force.**—To explain the distribution of energy throughout the field, the kinetic hypotheses suppose the existence of cyclic motions, which take place about axes coincident with the lines of force, the positive sense along each axis of rotation being specified in accordance with the convention of § 121.

If we look along the axis of a concentric system of lines of force in the direction from + to −, the lines appear to encircle it in the clockwise direction; and since the rotational motion around a line of force is assumed to follow the same law, those particles of an elementary tube of force which are nearest the axis must be moving in the sense from + to —.

Fig. 49 represents diagrammatically a longitudinal section of the field. S is the current-conductor, rectilinear in the case now considered. The four circles r_1 to r_4 above and below represent the cross-sections of magnetic tubes of force surrounding the wire S. If the direction of the axis of force S is from left to right in the figure, we must think of the magnetic lines of force as passing away from us as they cut through the plane of the paper above S, and coming towards us as they cut through this plane below S. In other words, when we look at the upper row of circles we are looking in the positive direction of the lines of force, and when we are looking at the lower row the contrary is the case. Around the upper circles, then (as indicated by the arrows), the rotational motion is clockwise, and around the lower circles, counterclockwise. Since the existence of tubes of force can be traced right up to the bounding surface of the wire, the motion taking place in the neighbouring parts of the field must be in the sense from + to — (or K to Z), as the arrows indicate.

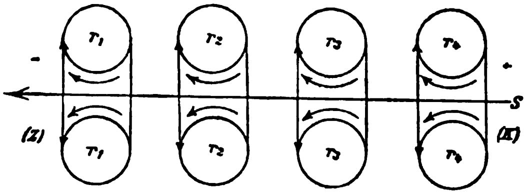

Fig. 49

These relations may be very simply exhibited on our apparatus (fig. 47); a number of closed tubes of force (§ 127) being placed one over another, embracing the wire L, which constitutes their common axis. If we have arranged the rings so that their cyclic direction properly corresponds to the red arrow on the wire (compare § 147), the direction of the motion in question is seen to be as stated.

We are thus provided with a simple diagram of the motion in an axial field of force; it is always easily reproduced, and expresses very conveniently the relations and rules which we have to remember. The motions in the immediate neighbourhood of an axis of force thus resemble a series of vortex rings closely following one upon another, and moving in the same direction.

Systems of coaxal smoke rings.—With the smoke-ring apparatus (§ 185), by striking a number of blows upon the end in rapid succession, a whole series of vortex rings, having the same axis, may be produced. If we strike the apparatus in the rhythm ⏑ - *i.e.* a weak stroke followed by a strong one, we obtain first a ring which moves slowly forward, and then one which follows more rapidly. If the motion in the second ring is too considerable, it produces a disturbance of the first, but otherwise it is easy to make the second more rapidly moving ring overtake the first and still remain concentric with it. It will then be noticed:

(*a*) That each of the two rings preserves its individuality. If the impulses given to the rings are quite axial, each will preserve throughout its circular form. In no case is there any intermixture of the two vortically moving portions of air, the vortex filaments remaining quite distinct from one another.

(*b*) If the second ring is sent strongly enough in pursuit of the first, this latter will expand so as to allow the second ring to pass through, afterwards closing up again. This passage of one vortex ring through another shows very clearly how such cyclic motions can go on side by side in space.

B.—Circular currents

There would seem to be a great difference between the fields of force considered in Section I. and the concentric system of circular lines surrounding a straight current-conductor. But the means of investigation already employed—iron filings, movable poles, small exploring magnetic needles—show that in the field surrounding a current we are dealing with magnetic lines of force pure and simple. To bring these fields into closer relation with those already considered we must depart from the simplest geometrical form of the conductor (rectilinear), and pass on to the investigation of more complicated forms. These we may suppose to be produced by suitably bending the conductor, the distribution of lines of force being correspondingly distorted.

149. Changes of form in a concentric system of lines of force, produced by bending the axis.—Let a wire, originally straight, and constituting an axis of force, be bent into the form of a circle, so that two portions of the wire which were originally far apart are brought quite close together, the direction in which they run being at the same time reversed. The lines of force embracing the various parts of the axis will thus be collected into a bundle, which passes through the nearly closed loop. Each of the lines of force retains always the form of a closed curve, whether the conductor itself be straight or curved. In the neighbourhood of the wire, where the force is greatest, and where (in accordance with our previous experiments) the tubes of force are richest in energy, the lines preserve most nearly their circular form. But since, in bending the conductor into a loop, we have collected into a bundle lines of force that were originally spread far apart, it follows that the tubes of induction of smaller energy-content are deformed in such a way that in passing through the loop these are crowded more closely together, while outside the loop they are spread further apart.

(*a*) *Line-of-force diagram for a current flowing round a ring.* To show by means of iron filings the course of the lines of force within and without the ring, bend a thick copper wire into the form of a semicircle, thrust its two extremities through corresponding holes in a horizontally placed sheet of paper, and then bend them round until they come almost, but not quite, into contact, so that the wire takes the form of a nearly complete circle (fig. 50). When a current is sent round the wire in the direction (+ to −) indicated by the arrows, the general course of the lines of force is as shown diagrammatically in fig. 50. Fig. 51 shows the actual disposition of the filings.

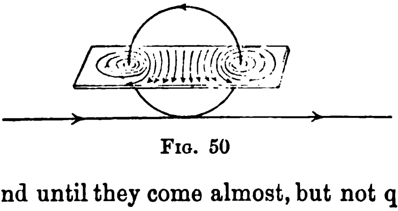

FIG. 50

The direction in which the lines of force are to be positively reckoned may be easily deduced from the positive direction along the conductor (axis of force). If this latter direction is counterclockwise from our point of view, the lines of force will encircle the left limb of the wire clockwise as seen from above. These lines, then, as they thread *through* the ring are directed towards the observer, the same being true of the lines which encircle the right limb of the wire, and, in fact, of all lines of the system.

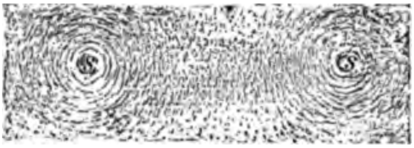

FIG. 51

[Thus, a translation in the direction of the magnetic force, combined with a rotation in the sense of the current, would constitute a right-handed screw motion. Comparing this statement with the corresponding square-bracketed statement in § 147, we may say that a current and the lines of induction which it produces embrace one another right-handedly.]

The relation between the cyclic directions of currents and their lines of force may be simply exemplified as follows:—Bring the thumb and forefinger together in each hand so as to form two closed circuits, which are made to embrace one another like consecutive links of a chain. The forefinger being considered in each case as pointing along the positive direction of the circuit, the forefinger of the right hand should be pointing downward through the left-hand circuit. This being so, the right-hand circuit may be taken to represent the current, and the left-hand circuit the lines of force, or *vice versâ*.

(*b*) *Model of tubes of force for a current in a ring.*—Over a copper wire SS, fig. 52, are threaded a series of closed rings, representing tubes of force, all of the same size, and corresponding in cyclic direction to the + to − direction of the current. The wire being then bent into the form of a closed or nearly closed circular loop, the rings must be arranged so that their planes converge towards the axis of the loop, as indicated in the figure. The arrows show that the hypothetical vortical motions within the conductor are everywhere in the same sense, namely in the sense of the axis of force (sense of the current).

The model, fig. 52, with the rotatory motions which it indicates, furnishes a chart of what occurs in the field of a circular current, according to the kinetic hypothesis; it may also be taken to exemplify the structure of a 'vortex sponge,' such as English mathematicians have elaborated from the kinetic theory. Although these motions may appear to be very complicated, their relations are very easily made out, for all the separate motions are determinately related to the axis of force. Moreover the model shows clearly the distribution of energy in the neighbourhood of the wire.

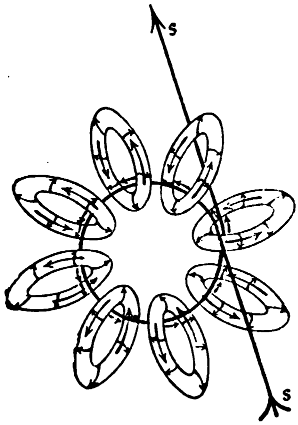

FIG. 52

150. A circuital current as a store of magnetic field energy.—Each tube of induction, or as we may say when the permeability of the medium is unity, each tube of force, is the seat of some definite amount of energy (compare § 119). If now, by bending a current conductor into the form of a loop, we compress into a limited compass tubes of force which were originally spread wide apart, we are thus accumulating a certain amount of energy, somewhat as we accumulate energy in a weight when we raise it against gravity.

A certain amount of work must be expended in the operation, for we have to overcome by a muscular effort the pressure exerted upon one another by the energy tubes in directions perpendicular to their length. [It should be remarked, however, that the more work is expended in this way, the smaller the magnetic energy becomes; the explanation being that work is meanwhile done upon the source of the galvanic current.]

That such pressure really exists in the tubes of force enclosing the current conductor, and gives rise to mechanical forces upon the conductor itself, may be easily verified experimentally.

Experiment 54.—A short, narrow, thin strip of gold has its ends joined to wires conveying a current. When these wires are held close together, the freely-hanging loop of gold, each time that the current is made to flow, is seen to bulge out at the sides. It strives to attain to the circular form, which, as we know, has the greatest possible area corresponding to a given perimeter. The lines of force passing through a circularly bent current conductor exert an outward thrust on the conductor itself, just as a confined mass of gas exerts a pressure on the walls of the containing vessel.

A current with its system of lines of force constitutes a store of magnetic field energy. The energy is much greater (for a given value of the current) when the space, or a part of the space surrounding the conductor, is occupied by a medium of high magnetic permeability, as for example when the conductor surrounds a mass of soft iron. Later on we shall have to speak more particularly of these important relations; for the present this preliminary note must suffice.

151. A circuital current as the equivalent of a magnetic shell.—When the axis of a concentric system of lines of force is bent into a circular form, we obtain something of the nature of a magnetic shell, whose surface is bounded by the current conductor. For the circuit, like the magnetic shell, emits lines of force in one direction, and these bend round, and return to it from the opposite direction. One side behaves like the north-seeking (red) face of the model described in § 23, the other like the south-seeking (blue) face. From a comparison of figs. 50 and 52 we obtain the following rule:

When we look at a circuit in such a direction that the current appears to flow in it counter-clockwise, the lines of force will be threading through the circuit towards us, and the north surface of the equivalent magnetic shell will be facing us; if, on the other hand, the current is so regarded that it appears clockwise, the lines of force will be threading through away from us, and we shall be turned towards the south surface of the equivalent shell.

Experiment 55.—To illustrate this law a thick aluminium wire is to be bent into a circular hoop, of the same diameter as the red and blue cardboard disc mentioned in § 23, and suspended by two gold strips about a metre in length. The aluminium ring has then but little freedom to move, except directly backwards and forwards; it cannot so easily rotate, and the mode of suspension does not allow it to move sideways. The ends of the wire are to be arranged crossing one another, but not quite touching, so that if we follow the wire along its whole length we pass first along a tangent to a circle, then around the circle itself, and, finally, along a continuation of the same tangent, the distance between the extremities of the wire being equal to the diameter of the circle. In these extremities saw-cuts are made, in which the ends of the gold strips are fastened. When the ring is suspended, a pin is placed underneath to mark its position of rest. If the upper ends of the suspending strips are now joined through a commutator to a pair of terminals, the freely-movable ring becomes associated with a bundle of lines of force. In each case the cardboard disc may be arranged so as to indicate the polarity of the circuit. On bringing near the north or south pole of a bar magnet, we observe the same phenomena of attraction and repulsion which were described in § 33.

If the extremities of a circularly bent wire are furnished with points which are bent downwards, and dip into mercury cups placed one below the other, we may then connect these mercury cups to terminals, so that a current flows round the wire (AMPÈRE'S suspension). Slight impurities at the surface of the mercury, however, interfere with the free motion of the system.

For purposes of comparison, and in view of the applications which we shall afterwards have to make, let us imagine a model to be constructed representing a circuital axis of force (circular current) with a series of tubes of induction passing through it (fig. 53).

A number of model tubes of force, appropriately marked with arrows, are collected into a bundle which passes through a single loop of the copper wire SS. If the tubes of force are bent into curves so as to follow the course of the lines of force, the resulting model resembles very closely that which was made to represent a magnetic shell (§ 128). The circularly bent axis of force is seen foreshortened in the figure, and when viewed from above follows the counter-clockwise direction of rotation. Thus, the observer looking from the point of view of the figure sees the wire pass from left to right between him and the tubes, and from right to left behind the tubes.

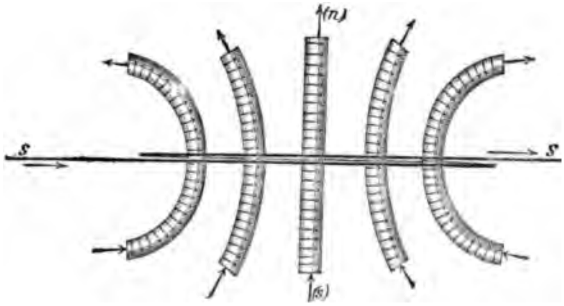

FIG. 53

C.—Conception of a current

From the consideration of the circuital axis of force as a centric system of magnetic lines of force, we are led to the conception of a current. We have to show how our mode of representing the phenomena leads to the conception of a progressive motion along the axis of force itself, that is, to a motion in the same sense as the elementary motions within the conductor. The most ordinary and familiar conception is of something moving along or even within the conductor, that is, in our example, along or within the wire forming the circuit. We say that there is a 'current' circulating in the wire. In order to have this, the circulation in the conductor, related in a definite manner to some determinate cyclic direction, we may make use of a transformation which is constantly employed in physics, and may appropriately be called AMPÈRE's transformation, having been first suggested by AMPÈRE. We shall consider it in relation to the model, fig. 53.

152. **Ampère's transformation.**—When a surface is bounded

by a line which nowhere intersects itself, and is completely filled with rotational motions all in the same sense (fig. 54), it may appear to a distant observer as if the motion, instead of being extended over the surface, were entirely confined to the boundary. For in the interior each element of surface filled with rotational motion is surrounded by elements in which the motion is similar. Hence it follows that in each portion of the bounding line of such an element motions in opposite senses are taking place; compare, for example, the adjacent surface-elements f_1 and f_2 in the figure. The motions are of equal intensity; that is to say, if equal velocities correspond throughout to equal surface-elements, there will be a mutual destruction of the external influences, due to the contrary motions along a line separating any two such elements. It is only the outermost surface-elements which are effective; for along those portions of their bounding lines which form part of the boundary UU of the whole surface no compensation takes place, since on the further side of this boundary (indicated by a heavy line in the figure) there are no adjacent elements. The compensation is complete when the circuits within which the rotatory motions take place are very small, as in the present case, where we are dealing with the cross-sections of tubes of force. Of the whole system of motion, then, the only part which is effective is confined to the boundary of the surface. This causes it to appear not as if any motion took place within the boundary (*i.e.* over the surface contained by it), but as if something were moving along, or 'flowing in,' the boundary itself.

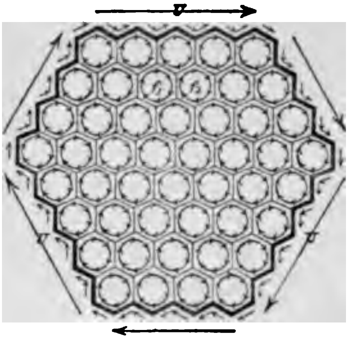

Fig. 54

This equivalence between a process extended over a bounded surface and a process confined to the boundary of that surface is a special case of the mathematical proposition known as STOKES's theorem. This theorem, by means of a general transformation, allows the sum of certain quantities connected with the separate elements of a surface, and satisfying certain conditions, to be expressed as the sum of certain other quantities, corresponding to the separate elements of the line bounding the surface. Thus STOKES's theorem enables us to prove rigorously that twice the sum of all the elements of a surface, each multiplied by a rotational velocity about an axis perpendicular to the element, is equal to the sum of the separate elements of the bounding line, each multiplied by the projection in its own direction of a certain linear velocity. This result is of great importance in connection with those theories which explain the field phenomena of magnets and currents by means of vortex motion. Ampère's transformation is the simplest case of the more general analytical theorem of which we have just made mention.

153. **Direction of the current; nomenclature.**—The cyclic motions which the kinetic theory supposes to exist within the compass of a circuital axis of force are equivalent to a progressive motion along the conductor itself. This motion along the conductor we may call a current, speaking similarly of a current conductor and a current loop. In accordance with AMPÈRE's transformation, the direction of this current must coincide with that of the elementary motions adjacent to the conductor; but this is the direction which we have already chosen for the axis of force, namely, from K to Z or from + to —, as shown in figs. 49, 52, 53.

We defined the positive sense along the conductor in this way when we gave distinctive names to the 'terminals.' We now find our convention to be justified, since Ampère's transformation shows that the sense from + to — along the axis of force corresponds to the direction of motion along the boundary of a surface enclosed by the conductor. Accordingly, the direction in which a current flows is determined without ambiguity by its magnetic properties. Since we have adopted in Chapter II. a definite convention as to the positive sense along the lines of force, the relation of a current to its magnetic field is completely determinate, nothing further remaining arbitrary.

Thus the whole system of conceptions is simply and definitely connected. It is only the nomenclature which presents any special difficulty. According to our view there is nothing flowing actually in the wire; there is, indeed, nothing taking place specially in the wire itself; all the processes with which we are concerned take place essentially in the surrounding space, that is to say, in the field. But all such terms as current, direction of the current, conductor, and so on (with which we cannot dispense, since we have no intention of introducing an entirely new nomenclature), are based upon the inadequate mechanical conception of a certain something flowing along the wire.

None of our conceptions have undergone so complete a change as have those relating to the phenomena of currents. There is, accordingly, much to be said for the many attempts made by English physicists to introduce entirely new names along with the new conceptions. Our object, however, is to show how known phenomena are to be interpreted and understood from the new standpoint, and this justifies us in retaining the old nomenclature.

154. **The field of force of a circuital current, and its image by reflection.**—We shall now proceed to give the proof of the axial character of the magnetic field-intensity, to which we have already alluded in § 120. The current flowing along a conducting circuit is only the boundary phenomenon of the system of lines of force which thread through the circuit. The two phenomena are mutually related, so that either implies the other. In accordance with a general law of symmetry of unbounded media, causes and their effects must possess the same type of symmetry.

Let us suppose now that a closed circuit in which a current is flowing is placed in a plane, each side of which behaves as a mirror. Then all points in the space above the circuit will appear by reflection as points in the space below, and, conversely, all points below will by reflection appear above. We may likewise suppose all the processes occurring in the actual field to be reflected so as to give an image of the field, extending on both sides of the reflecting surface. We have now to examine how the simple motions which we assume to exist in the field are affected by this process of reflection, and hence to draw some conclusion as to their symmetrical properties. The phenomenon of the current itself is evidently unchanged by reflection, because, according to our assumptions, it takes place in the plane of the mirror itself. The corresponding bundle of lines of force, on the other hand, is reversed in direction. If in the actual field the lines of force thread through the circuit from below upwards, in the reflected field they will thread through downwards from above. If, then, they possess the simplest kind of vectorial character, namely, that of a translation or flowing of something along them, a determinate direction of this flow (from below upwards, or the reverse) cannot be unalterably associated with a determinate cyclic direction of the current; for this latter remains unchanged on reflection, while the direction of flow along the lines is reversed. But such a conclusion is contradicted by experience. For a determinate direction of the current always corresponds to a determinate direction of the lines of force, and, conversely, when the direction of the current is given, the magnetic polarity of the circuit is determinate (§ 151). Thus the vectorial process or arrangement which conditions the existence of the lines of force must be something different from a simple flow or translation along the lines.

Let us now apply the same process of reflection to rotational movements about the lines of force as axes. Rotations in planes parallel to the plane of the mirror are not reversed on reflection, as is immediately evident. Rotations about axes oblique to the mirror plane are transformed by reflection into rotations belonging to the *same* line of force (compare fig. 53). Motions associated with the lines of force in the actual field, and having the same cyclic direction as the current in the conductor, fulfil the same condition in the reflected field. Thus these rotational motions are affected in the same way by reflection as is the current itself. Whatever the physical constitution of a line of force may be, it is due to something possessing the same type of symmetry as a rotation about an axis. This is important evidence of the 'axial character of magnetic lines of force,' to which we have already alluded.

We shall find our conclusions verified if we consider what takes place on reflection of the whole field at a mirror plane, passing through the axis of the current conductor, and, consequently, perpendicular to the plane of the latter. The sense of the current, flowing perpendicularly to the mirror, is reversed; while if we suppose something to be simply flowing along the lines of force, its direction of flow is not reversed. On the other hand, a clockwise rotation in a plane perpendicular to the mirror will be changed by reflection into a counter-clockwise rotation. Thus the boundary phenomenon of the current along the conductor possesses the same type of symmetry as the bundle of lines of force passing through it when, and only when, the lines are supposed to be axial in character.

A magnetic field is not reversed in direction by reflection in a plane mirror perpendicular to its lines of force; on the other hand, it is reversed in direction by reflection in a plane passing through its lines of force.

Curie expresses this result very suggestively by saying that magnetic lines of force possess the symmetry of an axis of rotation, not the symmetry of an arrow. If we make use of the symbols which are ordinarily employed in crystallography, the expression for the symmetry of a magnetic field of force will be

$$\frac{L_\infty \, l_\infty}{P_\infty} C.$$

155. **The galvanic current as a bundle of lines of force.**— However far we follow the course of a conductor conveying a current, we find it associated with magnetic lines of force. Hence we conclude that:

Every galvanic current is equivalent to a bundle of lines of force. So far as external magnetic effects are concerned, every circuit in which a current is flowing may be replaced by an equivalent magnetic shell, whose boundary coincides with the path of the current. Thus the current as a whole, that is to say, the entire circuit round which the current flows, may be regarded as equivalent to a magnet.

Ampère developed this equivalence from the opposite point of view, his aim being to explain all magnetic phenomena in terms of currents. Since a current flowing round a circuit may replace an equivalent magnetic shell, we may suppose a magnet to be built up of currents flowing round minute (molecular) circuits. This conception was successfully developed; it was the first explanation of magnetic phenomena in terms of the galvanic current. We must examine the relations between currents and magnets somewhat closely, as we shall have to make use of the converse transformation.

Although the behaviour of currents and magnetic shells is in many respects identical, yet there are fundamental distinctions which must not be overlooked; and these we now proceed to investigate.

D.—Differences between the fields of magnets and of galvanic currents

Without entering here into the more elaborate mathematical treatment of the question, we may direct our attention to certain peculiarities of the magnetic fields of currents and magnets, and the differences between them, adopting a mode of investigation which is diagrammatic rather than analytical. The 'field,' in our sense, will be the region external to any structure which exerts magnetic influence. Although the disposition of the lines of force which thread through a current circuit is in many respects similar to that observed in the case of a magnetic shell, yet there is one way in which the two fields differ fundamentally from one another: the field of the current continues undisturbed through the region embraced by the circuit; we can follow the course of the lines of force so as to pass from one side of the circuit to the other without being in any way hindered by the conductor itself (compare fig. 53); on the other hand, in the case of the magnetic shell we are stopped by the material of the shell, however thin it may be. Thus the difference appertains to the spacial distribution of the field.

156. **Simply- and multiply-connected regions.**—We shall now consider two magnetic fields; on the one hand, the field or space external to a plate of magnetic material (magnetised transversely) and, on the other hand, the field of a wire bent into a closed circuit, and traversed by a current. In order to make the difference in question as evident as possible, let us think of each field as bounded by an outer envelope which does not intersect the magnetic shell or circuit—for example, the walls, floor and ceiling of the room. If across the region we extend a diaphragm, which reaches as far as the outer envelope in all directions (its bounding line lying entirely upon the envelope), the region, if of ordinary form, will be thus divided into two entirely distinct and separate parts, each of which is completely bounded by a portion of the original envelope and one side of the interposed diaphragm. Such regions are called 'simply connected' (LISTING, *Census räumlicher Gebilde*). If two or more diaphragms are needed to effect this partition of the region, the latter is said to be 'doubly connected' or 'multiply connected.' The magnetic field due to a current in a circuit, in contradistinction to the field of an ordinary magnet, occupies a doubly-connected region. To realise this we must next consider:

(*a*) *The spacial connection of the free field of an ordinary magnet*, choosing for the sake of closer similarity a flat magnet or magnetic shell L (fig. 55a). Theoretically, the field extends from the external surface of the magnet L to infinity; practically, we may suppose it limited by some easily specified envelope, such as the walls of the room (in which term we include the floor and ceiling), represented in the figure by A. The region occupied by the field has thus the form of a hollow shell, bounded externally by the envelope A, and internally by the surface of L. A diaphragm SS of arbitrary form is now stretched across this region, the bounding line or lines of the diaphragm lying entirely flecting on the bounding surface or surfaces of the region. SS extends from A inwards, and must terminate where it intersects the surface of L. In the figure the intersections with these surfaces are indicated by dotted lines. The diaphragm itself has the form of a flat ring, somewhat as in fig. 55b.

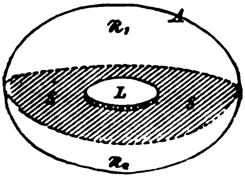

Fig. 55a

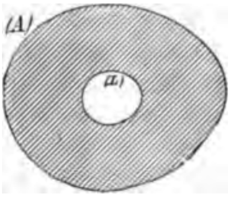

Fig. 55b

It divides the region occupied by the field into two entirely distinct and separate regions $\mathfrak{R}_1$ and $\mathfrak{R}_2$ (fig. 55a). We have now to consider :

(*b*) *The region external to a closed circuit* (*current conductor*).

A piece of rubber tube may serve as an illustrative model; its two ends being joined together by fitting them over a short wooden cylinder of suitable diameter.

The field extends from the surface G of the conductor (fig. 56a) to the walls of the room A. But now, however we may dispose the diaphragm SS, there remains a free communication between the regions $\mathfrak{R}_1$ and $\mathfrak{R}_2$, through the aperture of the ring (compare figs. 56a and 56b, where different forms of the diaphragm SS and the bounding surfaces A and G are represented in section).

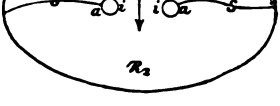

Fig. 56a

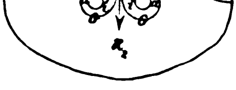

Fig. 56b

The bounding surfaces are, on the one hand, the surface of the ring G (including, of course, the portion *i* next the aperture as well as the more exterior portion *a*), and, on the other hand, the outer envelope A. If we extend a diaphragm from the envelope A to the bounding surface of G, we have once more a ring-shaped diaphragm as indicated in fig. 56c. The outer bounding line of the diaphragm lies on A, the inner boundary upon G. Since the surface S is to be a simple surface which nowhere intersects itself, it is evidently impossible by means of a single diaphragm of this kind to divide the space external to G into two entirely distinct regions. The partition may, however, be effected by introducing a second diaphragm, as indicated in fig. 56d. Here $\mathfrak{R}_1$ and $\mathfrak{R}_2$ are entirely separated from one another, each of these two regions being completely enclosed by a portion of the bounding surfaces A and G, and by one side of each of the two diaphragms, S_1, S_2.

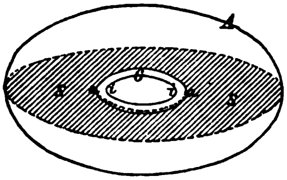

FIG. 56c

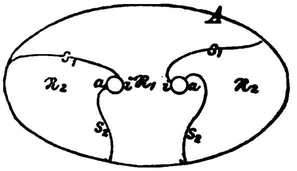

FIG. 56d

The field due to a current flowing in a circuit occupies a region in which there is a ring-shaped cavity (the space filled by the conductor), and as we have seen such a region is doubly connected.

It is this spacial property of the electro-magnetic field which renders it possible for the lines of force to bend round so as to form closed curves embracing the conductor, and lying entirely in the field, where no sources or sinks exist; neither diverging from poles (divergence = 0) nor converging to them (convergence = 0). In § 169 we shall have occasion to speak of other peculiarities of the electro-magnetic field arising from these spacial properties.

CHAPTER IX

MUTUAL ACTIONS OF CURRENTS AND MAGNETS

We have already made use of the mutual actions of galvanic currents and magnets in demonstrating the existence of lines of force due to a current, and investigating their disposition. Since the lines of force arising from a current behave exactly like those of a magnet, and only differ from them in the way in which they encircle an axis of force, it is natural to conclude that the two systems of lines of force will so influence one another as to produce mechanical forces between the current conductor and the magnet.

In this chapter we shall give some account of the most important cases in which a fixed current conductor exerts mechanical force upon a movable magnet, considering the phenomena first of all in their qualitative aspect, and proceeding afterwards to describe the principles of those methods of measurement by which such effects are brought into relation with the general system of units sketched above.

A.—Qualitative phenomena

As an introduction to the study of the mechanical forces due to electro-magnetic action we shall consider some simple examples which are of especial importance, partly from the point of view of history, partly on account of their applications. Here again we shall be guided by a study of the lines of force, and it is to the diagrams of these lines that we shall attach the greatest importance.

157. **A magnetic pole in the field of a current.**—The pole which we shall use will be one extremity of a long thin bar

magnet, and we shall investigate the case where such a pole is introduced into the field of a current. We shall take the plane of our diagram perpendicular to a straight vertical portion of the path of the current, fixing the bar magnet near to the conducting wire, and parallel to it.

Figs. 57 and 58 were obtained by means of the apparatus pictured in fig. 47, the end of a bar magnet being inserted into a hole in the board C, close to the aperture through which the conductor passes, and so contrived that the magnet was held vertical, that is, parallel to the direction of the current.

Fig. 57 shows the deformation produced in a field of concentric magnetic lines of force by the introduction of a magnetic pole, *i.e.* the south pole *s*. The current S passes through the plane of the diagram from below upwards. The figure may be compared with fig. 48, which represents the undisturbed field. Of the lines of force which densely encircle the cross-section S, a considerable number end at the 'sink' *s*. Here they leave the open field and pass into the substance of the magnetised steel bar, a place of no magnetic force making its appearance just above *s* (compare § 34). Beyond the range of the sink *s* the lines of force bend round in wide curves, passing through the undisturbed side of the field and completely encircling the conductor. Remembering that there is a tension along the lines of force, tending to draw together the bodies on which they terminate, we see from an inspection of the figure that the pole *s* must be acted upon by a force lying in the plane of the paper and perpendicular to the line joining S*s*. The pole then will be urged along the (negative) direction of the lines of force—that is, from our point of view, in the direction of clock hands, around the conductor S.

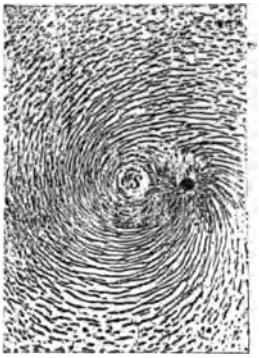
FIG. 57

If we bring into the field a sufficiently powerful source of lines of force, in the form of a north pole *m* (fig. 58), we see that the lines of force proceeding from this pole unite with a corresponding number of the lines encircling the conductor *S*, so that in a certain sense they now belong to the field of the current. The current is flowing in the downward direction, so that the lines of force pass clockwise round S.

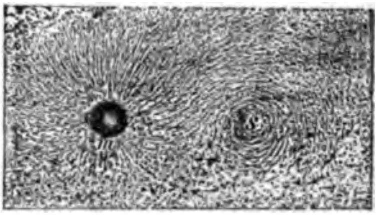

FIG. 58

Within the circuit of the lines proceeding from *m*, it will be seen that there are lines of force which pass completely round S, without interruption. There are also lines proceeding from *m* towards the lower part of the diagram, which nowhere unite with lines due to the current; they are bent aside, so that in the lower portion of the figure there is a space nearly free from lines of force. Hence the pole *m* is urged towards the upper part of the figure, *i.e.* clockwise around the conductor (§ 147). The two figures 57 and 58 show that magnetic lines of force are essentially of the same kind. They merge into one another, uniting together to form a resultant field, just as the lines of force from two permanent magnets would do.

158. **Mechanical forces experienced by a movable magnetic pole in the field of a current.**—In accordance with the line-of-force diagrams, figs. 57 and 58, a freely movable magnetic pole placed in the field of a current must tend to move continuously round and round the current-conductor. This tendency is in fact exhibited by the pole of the bar magnet mentioned in § 147, though a complete journey around the wire could not be made, since the body of the magnet was stopped by coming into contact with the upper side of the wire rectangle. Arrangements have, however, been contrived which enable magnetic poles to revolve uninterruptedly round fixed current-conductors. But in all such

cases the path of the current between two sam-eway directed magnets (used for the sake of symmetry) is bent at right angles, and is ordinarily made to dip into a circular groove filled with mercury, from which a fixed conductor leads the current away. This last it is, then, which produces the effect in question. A continuous rotation, due to concentric circular lines of force acting upon magnetic poles movable along the direction of these lines, is no more realisable than is a single isolated magnetic pole, dissociated from the pole of contrary sign.[1]

It is only by using a tube as the current-conductor, and placing within the tube like poles of suitably bent magnets, that we can produce a continuous rotation of the other poles, which are situated in the field external to the magnet.

159. **Magnetic needle parallel to a current-conductor; Oersted's fundamental experiment.**—We must next consider the behaviour of two unlike poles which are present simultaneously in the field of a current, at the same distance from the conductor, and to this end we shall make use of the two poles of the same magnetic needle. Parallel to the needle we hold a portion of the current-conductor, stretched straight between our hands, as in OERSTED's experiment:

Experiment 56.—A straight portion of a wire conveying a current is held horizontally in the magnetic meridian, exactly over a declination needle to which it is gradually approached. Thus the direction of the wire is parallel to the original direction of the needle, but as the wire is brought nearer the needle gradually deviates from its original north and south direction.

Next let the conductor be brought in the same way near to the needle, but from below instead of from above. The needle is again deflected, but in the opposite sense.

The line-of-force diagram corresponding to OERSTED's experiment is shown in fig. 59.

Upon the thick conducting wire used in § 145 a little bar magnet is fastened with sealing-wax, its axis being parallel to that of the wire, and over all a sheet of paper is laid.

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[1] E. Lecher, *Weid. Ann.* 54, p. 282. 1895.

At S, S, towards the edges of the diagram, where the current alone exerts an appreciable influence, short, nearly straight, transverse bands of filings will be observed, these being due to the lines of force which embrace the conductor concentrically. They have a determinate direction, which in the present case is from the lower towards the upper part of the figure (notice the arrows by S, S, giving the direction of the current). But at those parts of the field of the current which are within the range of influence of the little bar magnet, the transverse bands of filings are greatly displaced. Those lines of force of the current which approach the north pole from the lower part of the figure are bent aside, so that the space just below *n* is traversed by very few lines, and the filings are subject to no appreciable directing influence. On the other hand, the lines of force proceeding from *n* unite with those lines in the field of the current whose general direction is the same (*i.e.* towards the upper part of the diagram). It will be seen how lines of force proceeding upwards from *n* pass into the undisturbed current-lines at the right-hand end of the bar.

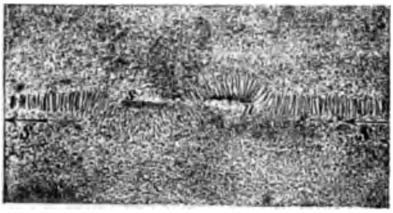
FIG. 59

At the other end of the magnet, all these relations are reversed. The lines of force proceeding upward from the lower part of the diagram are absorbed by the south pole *s*, which behaves as a sink, where they disappear; while the region just above *s* is free from lines of force. If we imagine the little bar magnet to have freedom of rotation in a plane parallel to the axis of the wire, the tension along the lines of force will cause the north pole to be urged towards the upper part of the diagram, and the south pole in the contrary direction. We see, accordingly, how the deflection of the needle observed in experiment 56 is conditioned by the course of the lines of force.

160. The thumb rule; Ampère's swimming rule.—The line-of-force diagram, fig. 59, shows further in which direction the deflection of the needle takes place. If we wish to have some simple rule connecting this direction with the direction of the current, we may conveniently make use of the disposition of certain parts of the body. For example, if we suppose the right hand to be laid upon the wire, so that the direction of the current is from the wrist towards the middle finger, the palm of the hand being turned towards the magnetic needle, then the north pole of the needle will be deflected in the direction of the thumb. In fig. 59, the hand must be held pointing from right (wrist) to left (middle finger), the back of the hand being turned downwards, since the wire passes beneath the needle.

Since the lines of force due to the current encircle the wire in the clockwise direction, while the north pole of a magnetic needle tends to move along them in the positive sense, the north pole must always be deflected in the direction indicated by the thumb; in the present case, towards the upper part of the figure. This rule we shall call the 'thumb rule;' it must be remembered that it refers to the *right* hand and the *north* pole of the needle. As regards the south pole, we have only to remember that its behaviour is exactly opposite to that of the north pole.

Another rule, due to AMPÈRE, is known by the name of the swimming rule. It supposes the whole body brought into the path of the current, and is stated as follows: Imagine yourself to be swimming in the conductor in the direction of the current, with your face turned towards the needle; then will the north pole be deflected towards the left and the south pole towards the right. In fig. 59, for example, the observer must be lying on his back, with his head to the left and his feet to the right, and looking up at the line-of-force diagram above him. The north pole of the magnetic needle will then be deflected towards his left, that is towards the upper part of the diagram, and the south pole in the contrary direction. In connection with AMPÈRE's rule, we have to remember 'left hand = north pole.'

This swimming rule has come to be very extensively used; but we prefer the thumb rule, since the conception of the whole body in a position which may be highly uncomfortable is not so convenient as the employment of the hand alone. To apply Ampère's rule to a dynamo machine in motion might indeed be dangerous to life, for it is well known that in the effort to imagine the body in any particular position, we often attempt in some degree to practically realise our conception, and this might lead to an entanglement with driving belts, cranks and shafts.

161. **Galvanoscope.**—The deflection which a magnetic needle suffers owing to a current flowing in a conductor parallel to its position of rest has been made the basis of a number of instruments which serve to detect the existence of a current. If a current is brought close to a magnetic needle which can turn freely in a horizontal or in a vertical plane, a deflection in one direction or the other will result, and will serve to show in which sense the current flows. The magnitude of the deflection will also furnish some idea of the number of lines of force arising from the action of the current, for it is immediately evident that the greater the number of these lines taking part in the phenomenon, the greater (*cæteris paribus*) the deflection of the needle will be.

162. **Two magnetic poles in a plane perpendicular to the current.**—A magnetic needle must also be deflected when a vertical portion of a current-conductor, lying in the same magnetic meridian as the middle point of the needle, is brought near to either its north or its south pole.

Experiment 57.—Let a portion of a current-conductor, stretched straight and held in a vertical position, be brought from the north or from the south towards the corresponding pole of a declination needle, supported on a high stand, the direction of the current in the conductor being indicated by arrows. A deflection of the needle will take place, corresponding to the direction of the lines of force.

Here again the thumb rule, or Ampère's rule, may be applied.

The line-of-force diagram for this case is shown in fig. 60.

In a longitudinal groove in the board C, near to the conductor (fig. 47), a little bar magnet is placed, its axis pointing directly towards that of the conductor; and in a plane just over the magnet the line-of-force diagram fig. 60 is formed.

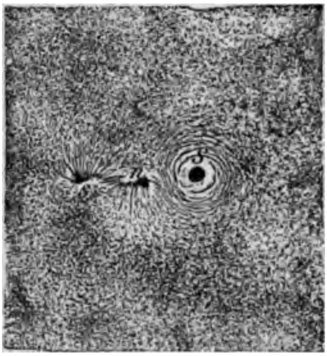

FIG. 60

Surrounding the cross-section S of the conductor is the concentric system of rings due to the lines of force of the current (compare the diagram of the undisturbed field, fig. 48). At the side where the magnet is, with its source *n* and sink *s*, the two fields are superposed. The current in the present case is flowing upwards from below through the plane of the diagram, for the lines of force, as viewed from above, encircle the conductor counter-clockwise, as may be seen from their behaviour on encountering the poles *n* and *s*. If the little magnet is free to rotate about its middle point, it will accordingly be deflected (experiment 57). If the reader imagines his right hand to be thrust through the plane of the paper, the wrist being below and the fingers above (corresponding to the direction of the current), while the palm of the hand is turned towards the needle, then will the north pole be deflected in the direction in which the thumb points.

163. **Magnetic needle with one degree of freedom surrounded by a current loop.**—The effect of a current in deflecting a magnetic needle will be increased when the effects observed in experiments 56 and 57 are combined together. We may accordingly bend the current-conductor into the form of a nearly closed loop, surrounding the needle.

To obtain the line-of-force diagram in this case, we may make use of the following simple arrangement. Of two wooden blocks, each about 12 cm. long, 8 cm. broad, and 2 cm. thick, one is used to support the upper part of the current loop. Through this block, perpendicularly to its face, two holes are bored about 5 cm. apart, and through the holes a ∩-shaped piece of thick copper wire is passed, the block being covered with a sheet of paper in which there are corresponding holes. The first block is laid upon the second, and in this there are small hollows filled with mercury into which the ends of the wire dip; the mercury being connected to terminals by means of metallic conductors. The line-of-force diagram can be easily prepared, and fixed on removing the loop. When the loop is about double as long as it is broad, the diagram obtained is very nearly the same as if the circuit around the needle were completely closed.

In the line-of-force diagram fig. 61, the points marked c_1 and c_2 are the places where the conducting wire passes perpendicularly through the plane of the paper. At c_1 the current passes from below upwards, while at c_2 it passes downwards again. Accordingly at c_1 the lines of force embrace the conductor counter-clockwise, and clockwise at c_2 where we are looking at the figure along the direction of the current. Between the points c_1, c_2 lies the needle *ns*, as is shown by the disturbance of the lines of force. Two bundles of lines of force due to the current, proceeding in opposite directions, are attached to the bar magnet. In the middle, at the indifferent zone, the two bundles meet. The tension along the lines of force tends to turn the needle out of the plane of the current loop which passes through c_1, c_2, the north pole being urged towards the lower part of the diagram, the south pole towards the upper.

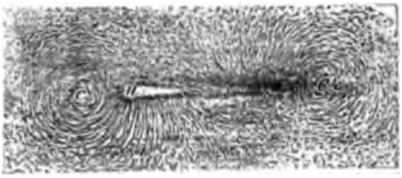
FIG. 61

If the right hand be laid upon the conductor in the sense in which the current flows (*i.e.* with the wrist at c_1 and the middle finger pointing towards c_2), and if the palm of the hand be turned towards the needle (*i.e.* downwards) the north pole *n* will be deflected in the direction of the thumb (*i.e.* towards the lower part of the diagram).

We shall see later that an arrangement similar to the above forms the basis of an important measuring instrument —the tangent galvanometer.

B.—Quantitative relations

That the field intensity at each point of an axial magnetic field may be determined in absolute measure, we must necessarily infer from Chapter IV., since the methods there given are of general application, and do not depend on the special properties of the field in question. It will be found, however, that for the field of force due to a current there is a single magnitude which serves to determine the whole field when the form of the conductor is given ; in the case of a straight conductor, for example, we may calculate the field-intensity for every point when we know its value for a single point. The magnitude to which we refer is the so-called 'current strength.' In introducing this important conception we would lay especial stress on the fact that it can be defined with reference to magnetic quantities alone, so that we do not need to consider any phenomena of a kind different from those which specially concern us. The remainder of this chapter will accordingly be devoted to a description of various methods for measuring 'current strength,' and an investigation of the energy which resides in the field due to a current.

164. **Fundamental electro-magnetic law.**—We have already learnt from an inspection of the line-of-force diagram in a plane perpendicular to a straight current-conductor (fig. 48) that the field-intensity in a coaxal magnetic system falls off as we pass to places more remote from the current. Biot and Savart made the first accurate investigation of the law according to which the decrease of intensity takes place, and they found that the field-intensity is inversely proportional to the distance from the conductor. This

result may be demonstrated very convincingly by means of a negative experiment. One or more bar magnets are supported in such a way that the whole system can turn about an axis coinciding with that of the rectilinear conductor (fig. 60).

No rotation in either sense is found to take place, whence it follows that the forces acting on the pole *n* (fig. 60), and those acting on the pole *s*, have equal and opposite moments about the axis in question; the moment of the forces on *n* being counter-clockwise, and on *s* clockwise. Thus in magnitude the force on *s* bears to the force on *n* the ratio $n\mathrm{S}/s\mathrm{S}$, as will be shown more rigorously later on.

The experiment may be performed in the following manner: AA, fig. 62, is a base-board, 40 cm. × 40 cm., standing on feet, and supporting a suitably stayed vertical stem BB, one metre in height. From the upper end of this stem projects a horizontal arm E, carrying a hook C, from which the ring *r* (foreshortened to a horizontal line in the figure) is suspended by a fine but strong thread. Projecting upwards from the ring is a hook, whose extremity is turned inwards, and is attached to the suspending thread from C, so that the point of attachment lies vertically above the centre of the ring. To the ring *r* three threads are attached which serve to support the circular wooden disc D, 22 cm. in diameter. The threads are attached to three brass eyes, inserted in D at equal intervals (compare fig. 63, which shows the disc D as seen from above). Through the middle of the disc is a circular hole *o*, 2 cm. in diameter. Three bar magnets m_1, m_2, m_3, of equal size, and equally strongly magnetised, are fixed to the disc by means of brass fittings screwed down to the wood. They all present like poles towards the hole *o*, and can rotate freely along with the disc about a vertical axis through the point of suspension of the ring *r*. The current-conductor EEE passes axially through the system of lines of force arising from these magnets. It starts from a terminal at the top of the stem B, passes along the horizontal arm, and is bent a little circuitously to enable it to pass centrally through the ring *r* and the hole *o* in D. It continues its straight course down to the base-board A, along which it passes horizontally to a second terminal. The system of magnets cannot make a complete revolution about its axis, because the wire attached to the ring *r* would be stopped on encountering the conductor, but there is quite sufficient freedom of motion to enable any considerable electro-magnetic action to be detected, supposing that the effects produced on the opposite poles of each magnet did not quite balance one another. To show that there is really a magnetic field produced by the current, although the system of magnets does not rotate, a small magnetic needle *ns* is supported

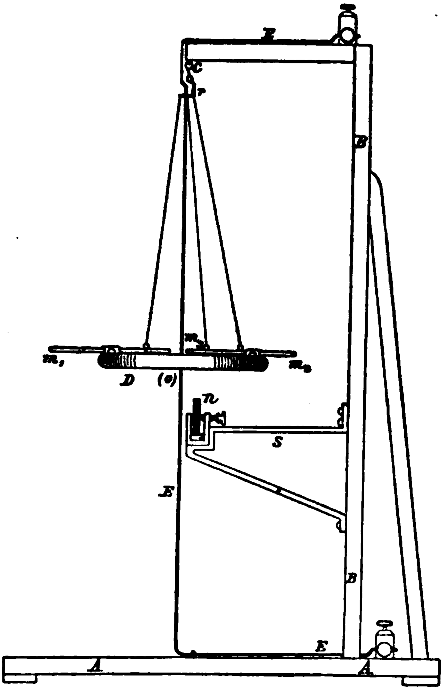

FIG. 62

at the end of the bracket S, being free to turn about a horizontal axis which lies in the same plane as the conductor E, the needle being so close to the conductor that it is considerably deflected as soon as the current circulates. The needle should be so small and so far from the magnets m_1, m_2, m_3 that it exerts no appreciable action upon them. (Through an oversight the needle *ns* in fig. 62 has been made somewhat too large, and too near the disc D.)

The behaviour of bar magnets placed in the field of a current as in fig. 63 furnishes all the data necessary for the deduction of the law of distribution of field-intensity.

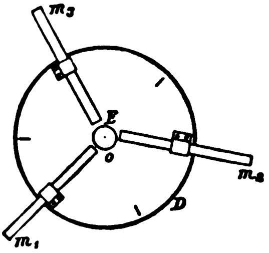

FIG. 63

Let us suppose the bar magnet m (fig. 64) to be free to rotate in the plane of the paper about a point S, so that its distance from this point is invariable, while its axis

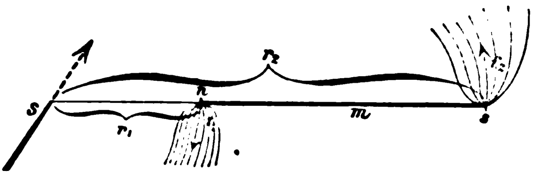

FIG. 64

always passes through the point. S is the point where the plane of the paper is intersected perpendicularly by a rectilinear current-conductor, whose direction is *from* the observer.

The lines of force in the field of the current encircle S in the clockwise direction. Let r_1 and r_2 be the distances of the two equal and opposite poles of the bar m from the axis of force S. The poles n and s are pulled in directions indicated by the arrows, n being urged towards the lower, and s towards the upper part of the diagram. If the strengths of the poles are $+m$ and $-m$, $\mathfrak{H}_1$ and $\mathfrak{H}_2$ the field-intensities at the places occupied by these poles, the force exerted on n is, by § 69, $f_1 = +m\mathfrak{H}_1$ dynes, the force on s being similarly $f_2 = -m\mathfrak{H}_2$ dynes, the signs + and − denoting that these forces act in contrary directions. We have, in effect, a lever whose fulcrum is at S, and which is actuated by the force f_1 at the end of the arm r_1 in one sense, and the force f_2 at the end of the arm r_2 in the contrary sense.

Such a lever is in equilibrium when the opposing moments, measured by the product of each force and the arm on which it acts (f_1r_1 and f_2r_2), are numerically equal to one another.

In the present case it is actually found that the two effects completely balance one another, so that equilibrium results, even the strongest currents we are able to employ producing no rotation of the movable system in either sense. Hence we must have

$$f_1r_1 = f_2r_2$$

or

$$m\mathfrak{H}_1r_1 = m\mathfrak{H}_2r_2$$

whence

$$\mathfrak{H}_1 : \mathfrak{H}_2 = r_2 : r_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

In the magnetic field due to a very long rectilinear portion of a current, the field-intensity is inversely proportional to the distance from the axis.

This law of decrease of field-intensity with increase of distance is usually expressed by writing

$$\mathfrak{H} = \frac{C}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27a)$$

where C is a constant.

In fig. 49 we gave an approximate representation of the motion in the neighbourhood of a current, taking account of the axial character of the magnetic lines of force; we now proceed to consider a model which at the same time expresses the variation of field-intensity with distance. The model is shown in fig. 65.

A board B, 70 cm. long and 10 cm. broad, carries at its ends two small vertical wooden supports t_1 and t_2 10 cm. in height, and between these is stretched a copper wire AC marked with arrows, p_1, p_2, p_3. At three places along the board vertical brass rods are fixed, and these serve to support the smaller inner wooden rings r_1, r_2, r_3 and the larger rings R_1, R_2, R_3, so that their axes of symmetry coincide with the axis of the wire AC. The outer rings have twice as great a radius as the inner, and half as great a cross-section. All the rings are marked with arrows (not distinguishable in the figure), one set pointing round the rings in the clockwise direction as seen from A, the other set embracing the cross-sections of the rings in the clockwise

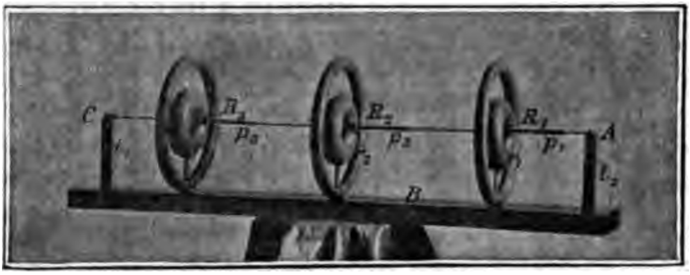

FIG. 65

direction as seen by an eye looking along the circumference of the rings in the direction indicated by the former set of arrows. The second set of arrows, when they reach the innermost part of the surface (*i.e.* when they are threading through the apertures of the rings) have the same direction as the arrows p_1, p_2, p_3 upon the wire AC. The cross-sections of the rings are made proportional to the field-intensities at the corresponding distances, that is, to the corresponding velocities in the rotational motion.

165. **The characteristic magnitude for the field of a current; conception of current-strength.**—In the equation (27*a*), $\mathfrak{H} = C/r$, the quantity C appears, for the measurement of which we have as yet no criterion; we only know that, for any given current-field of the kind considered, its value is constant. It may be resolved into two factors; for if we imagine a number of separate fields each due to a current flowing in a rectilinear conductor and all having their seat in the same medium, they will all be similar to one another, as may be inferred from the disposition of the lines of force. In each of them the field-intensity $\mathfrak{H}$ varies inversely as the distance r from the axis. But at a given distance r from

the axis the value of $\mathfrak{H}$ will in general be different for the different fields, depending on the value of the constant C. This constant accordingly involves a quantity characteristic of the special field considered, and we must now proceed to determine in what manner the value of C is related to this quantity. For two rectilinear current-conductors, for whose fields the constant C has the same value, the field-intensity $\mathfrak{H}$ corresponding to a given distance r will be the same. Suppose, then, that we bring two such conductors into close juxtaposition; the result will be an axial field formed by the superposition of two equal and similar fields. And since through each volume-element of the medium we have two systems of lines of force coinciding in direction with one another, their effects will be simply added together, as we saw in § 122. Thus by the superposition of the two original fields we obtain a new field at each point of which the field-intensity is twice as great, so that the constant C has twice its former value. Similar relations hold good when three or more such fields are superposed. We see, then, that C must be simply proportional to that magnitude which characterises the separate currents and the fields surrounding them. Denoting the magnitude in question by i, we may accordingly write

$$\mathrm{C} = \kappa i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27b)$$

where κ is an absolute constant. Provisionally we shall call i the 'characteristic of the current-field,' but we shall soon see that it is identical with the quantity called '*current-strength*' in the older theories of current phenomena. We have already agreed to retain such terms as 'current,' 'path of the current,' and others which are in common use, and we shall now introduce the term 'current-strength,' although the conception with which we associate it has nothing directly to do with the strength of a current.

Equation (27*a*) (Biot and Savart's law), when we introduce the quantity i, becomes

$$\mathfrak{H} = \kappa \frac{i}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The value of the numerical factor κ still remains undetermined; it evidently depends upon the permeability μ of the medium, since this latter quantity influences the number $\mathfrak{H}$ of lines of force per unit area corresponding to a given distance r under given circumstances. [Provided only that the medium surrounding the conductor is homogeneous and isotropic, the value of $\mathfrak{H}$ corresponding to any given value of r is entirely independent of the permeability μ.]

In most text-books the law of BIOT and SAVART is not given in this simple form, the phenomena being referred to the individual effects produced by infinitesimal 'current-elements.' If the force exerted on the unit pole $+1$ at the distance r from the rectilinear current-conductor is ascribed to the resultant action of the separate elements of the current, each of the elementary component actions must be proportional to the current-element idx which produces it, dx being an element of length of the conductor. The formula for the action of an element also contains as a factor the inverse square of the distance ρ, wherein lies a kind of formal analogy with the Newtonian law of gravitation; but we must also introduce into the expression a circular function which has no analogue in the ordinary theory of attractions. The field-intensity due to an element of current varies as the sine of the angle which ρ and dx make with one another, the complete expression being

$$\frac{idx \sin \alpha}{\rho^2}.$$

The correctness of this expression cannot be directly tested by experiment, since the existence of an isolated current-element is physically impossible. To obtain expressions applicable to practically realisable cases, we must first, according to the older methods, perform the inverse operation of adding together the effects due to all the elements of a circuit. This we will now do for the case of a long straight conductor, with which we have just been occupied, and we shall take the intersection of the line r with the conductor as the origin from which values of x are measured. Accordingly we have $\rho = \sqrt{(r^2+x^2)}$, $\sin \alpha = r/\rho$, and the expression to be integrated is

$$ir\frac{dx}{(r^2+x^2)^{\frac{3}{2}}}.$$

Since we suppose the rectilinear conductor to be indefinitely extended in both directions, the limits of integration will be $x = -\infty$ and $x = +\infty$, so that we finally obtain $\mathfrak{H}=2i/r$, a result identical with equation (28) provided we assume for the numerical factor κ the value 2.

In order to keep our system of units in agreement with that in common use, we shall so choose the unit for the current-strength i that the numerical factor κ assumes the special value 2. The law of distribution of field-intensity around a long straight conductor carrying a current of strength i must accordingly be written

$$\mathfrak{H} = 2\frac{i}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

[If the permeability μ of the surrounding medium is different from 1, equation (29) will still hold good, but we shall have $\mathfrak{B} = 2\mu i/r$, where $\mathfrak{B}$ is the magnetic induction.]

The above-mentioned method of resolving the electro-magnetic effect into elementary components, supposing it to hold good, enables us to calculate the distribution of lines of force at given points of the field, even when the current-conductor has not the simple rectilinear form. If, for example, we bend a portion of a conductor of length $2\pi R$ into the form of a plane circular loop, the number of lines of force per cm.^{2} at the centre of the circle may be deduced as follows: Every element of the ring is perpendicular to the radius R, so that for every element we have $\sin \alpha = 1$. The distance of the centre from each current-element is R, while the sum of the elements of length is $2\pi R$, and we obtain, therefore, for the force exerted upon a unit pole + 1 placed at the centre

$$\mathfrak{H} = \frac{2\pi R}{R^2}\,i = \frac{2\pi i}{R} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

166. **Dimensions of current-strength.**—Since by equation (29) $\mathfrak{H}=2i/r$ or $i=\frac{1}{2}\mathfrak{H}r$, the current-strength i is of the nature of a magnetic field-intensity multiplied by a length. We have seen, moreover, that the dimensions of $\mathfrak{H}$ are $[L]^{-\frac{1}{2}}\,[M]^{\frac{1}{2}}\,[T]^{-1}$, so that the dimensions of i are $[L]^{\frac{1}{2}}\,[M]^{\frac{1}{2}}\,[T]^{-1}$.

Since we have based our determination of current-strength on the electro-magnetic phenomena of the galvanic current, the system of measures implied in this last result is the electro-magnetic system, the current-strength i being measured in electro-magnetic units.

In defining the strength of a current we have only made use of the magnetic properties of the phenomenon, and on these the conception and evaluation of the current have been based. We therefore regard current-strength as a *purely magnetic quantity*.

The older conception was of something flowing along the conducting wire. If e measures the quantity of this something which has flowed past any cross-section of the wire since some definite instant of time, the current is to be measured by the increment of e per unit of time; that is

$$i=\frac{de}{dt}, \text{ or } de=idt.$$

The dimensions of the hypothetical entity measured by e are therefore

$$[L]^{\frac{1}{2}}\ [M]^{\frac{1}{2}}$$

in the electro-magnetic system.

167. Measure of a current in absolute units; the unit of current-strength; the Ampère.—Let part of the circuit in which the current is to be measured be in the form of a cylindrical wire, stretched straight in a horizontal direction (fig. 66). We suppose the current to be flowing towards the observer in this part of the conductor. Perpendicular to the direction of the wire, and lying in the same horizontal plane, is a thin bar magnet M, whose strength of pole m is known, and which is supported by means of two threads,

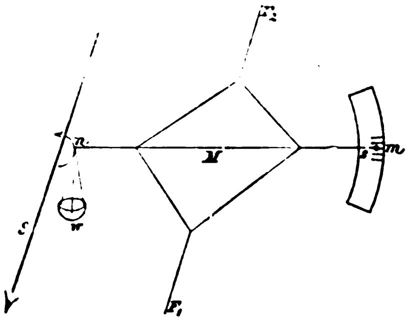

FIG. 66

F_1, F_2, of the form shown in the figure; the outer portions of F_1, F_2 being parallel to the wire, and allowing the magnet to turn freely and easily in a vertical plane. One end n of the bar magnet points towards the wire, to which it very nearly extends, and from it is hung a light scale-pan w, which so equilibrates the magnet that in its position of rest it points directly towards the wire. Close to the other end s of the magnet is a small scale with a fiducial mark m.

If a system of magnetic lines of force is produced by sending a current along S in the direction indicated by the arrow, the north pole n of the magnet will be urged upwards, and the south pole s will sink on the scale.

By placing weights in the scale-pan w, s may be made once more to rest opposite the mark m. If p grams are necessary to effect this, while m is the strength of the magnet pole, the strength of the magnetic field at the place occupied by n will be

$$\mathfrak{H} = \frac{981 . p}{m}.$$

We have now to measure the distance r of the pole n from the axis of the wire S, and dividing $\mathfrak{H}$ by this distance, we obtain $\mathfrak{H}_1 = \mathfrak{H} / r$ for the field-intensity at unit distance from the wire. The required current-strength in absolute measure is then

$$i = \tfrac{1}{2} \mathfrak{H}_1.$$

The slight disturbance of the field of the current arising from terrestrial magnetism may be neglected if the current is sufficiently strong; but in our method of measurement the result is uninfluenced by any effect due to terrestrial magnetism, whether great or small.

Using the bar mentioned in § 66, for which $m=2$ cm.$^{\frac{3}{2}}$ gr.$^{\frac{1}{2}}$ sec.$^{-1}$, and a stretched manganin wire 0·6 cm. in diameter, r was found to be 0·4 cm. and $p=0{\cdot}020$ gram. Thus the force exerted on the pole m was about 20 dynes, giving $\mathfrak{H}=10$ cm.$^{-\frac{1}{2}}$ gr.$^{\frac{1}{2}}$ sec.$^{-1}$ and $i=2$ in absolute measure.

The tenth part of the unit which we have used for the electro-magnetic measure of the current has been called after the French physicist Ampère. For example, referring to equation (29) we may say:

A current of one ampère produces at a distance of 1 *cm. from its axis a field-intensity of* 1/5 *of an absolute unit, supposing the permeability* μ *to have the value unity* [*or any other value*].

The absolute unit of current is thus equal to ten ampères, and is also in practical use. It is conveniently called a 'deca-ampère.'

The current measured above had accordingly a strength of 2 deca-ampères, or 20 ampères.

168. **The tangent galvanometer.**—The so-called '*tangent galvanometer*' provides us with a means of determining current-strength in absolute measure far more conveniently and certainly than would be possible with the apparatus described in the last paragraph. The instrument consists essentially of a plane circular ring which is brought into the magnetic meridian plane and is traversed by the current to be measured, while at its centre is a short declination needle, movable about a vertical axis. Fig. 51, p. 193, shows the line-of-force diagram for a horizontal plane through the position occupied by the needle. If R is the radius of the ring, the field-intensity at its centre is, by § 165,

$$\mathfrak{H} = \frac{2\pi i}{R}.$$

The region in the immediate neighbourhood of the centre of the ring is traversed by the lines of force with nearly uniform density, the directions of the lines being there perpendicular to the plane of the ring; that is to say, that around the centre of the ring the field of the current is nearly uniform. Each pole of the needle is acted upon by two forces, one due to the field-intensity $\mathfrak{H}$ and tending to deflect the needle away from the plane of the ring, the other due to the horizontal component H of the earth's magnetic field and tending to restore the needle to the aforesaid plane. In accordance with the principles for the composition of forces, there will be equilibrium when the needle is deflected from its original position through an angle ϕ such that

$$\tan \phi = \mathfrak{H}/\mathrm{H}.$$

We have, therefore, for the current-strength

$$i = \frac{\mathrm{RH}}{2\pi} \tan \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (31)$$

Since all the quantities on the right hand of the equation can be determined in absolute measure, i, in the same measure, will be expressed as a field-intensity H multiplied by a length R, in agreement with the result previously obtained for the case of a rectilinear conductor ($\tan \phi$ is, like 2π, a pure number without dimensions).

In the model, fig. 20, we may further illustrate the manner in which equilibrium is conditioned by a definite angle of deflection ϕ. For let another pair of threads similar to *ff* be attached to the hooks *hh*, and pulled horizontally in directions perpendicular to the line *rr* which represents the direction of the magnetic meridian. To effect this, the threads are to be passed over pulleys similar to *rr*, attached to blocks like k_1, k_2, and to have weights GG attached to their extremities. These weights represent the pair of forces (couple) exerted upon the poles by the field of the current while *gg* correspond to the forces arising from the horizontal component of the earth's magnetism.

169. **Work of electro-magnetic forces.**—The electro-magnetic forces acting upon a pole at a distance r from a rectilinear current-conductor would tend to move it round a circular path embracing the conductor. If the pole has the strength $+1$, while the current-strength is i cm. gr.$^{\frac{1}{2}}$ sec.$^{-1}$, the force exerted upon the pole will be numerically equal to $\mathfrak{H} = 2i/r$ dynes. If the pole is displaced through a distance dl in the direction of this force, the work done will be $d\mathrm{A} = \mathfrak{H} dl$. The element of length dl of the circular path of radius r may be expressed in terms of the elementary angle $d\beta$ subtended by dl at the centre of the circle, the relation being $dl = r d\beta$, so that $d\mathrm{A} = \quad r d\beta$. If the pole be allowed

to pass once completely round the conductor under the force exerted upon it in the field of the current, we have $\beta = 2\pi$, and the work done is

$$A = \mathfrak{H} . r 2\pi = \frac{2ir2\pi}{r}$$

or

$$A = 4\pi i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

This expression for A does not involve the distance r, so that the total work A will be the same in whatever part of the field the pole may be placed.

The path is the longer, the greater r is, but the force $\mathfrak{H}$ is smaller in the same proportion; and accordingly the work along an outlying line of force of the coaxal field is the same as that along one of the inner lines.

[If the permeability μ is different from 1, the work done is still $A = 4\pi i$.]

If the path of the pole $+1$ does not coincide with one of the lines of force, but has any other arbitrary form, the force doing work upon the pole along any element of the path will only be a component of the field-intensity $\mathfrak{H}$, the direction of this latter quantity being along the tangent to the line of force which passes through the position of the pole. The component in question will be less than $\mathfrak{H}$ in the same proportion in which the element of path is greater than the corresponding element of the line of force. Thus the work dA along the element of the arbitrarily chosen path is equal to that along the corresponding element dl of the line of force itself, in the case already considered, so that we have the work $A = 4\pi i$ corresponding to any circuit which once, and only once, embraces the current in the positive direction; the circuit may, for example, consist of the four sides of a rectangle.

The current-strength i may also be defined, then, in terms of the work done by magnetic forces upon a unit pole which passes once completely round the current-conductor. We have

$$i = \frac{1}{4\pi} A.$$

Here again it will be seen how the current-strength *i* may be regarded as a characteristic of the field.

Quantity of work per unit pole has in fact the same dimensions as those previously found for *i*, namely

$$\frac{[L]^{2}[M][T]^{-2}}{[L]^{\frac{3}{2}}[M]^{\frac{1}{2}}[T]^{-1}} = [L]^{\frac{1}{2}}[M]^{\frac{1}{2}}[T]^{-1}$$

If the path of the pole embraces the current more than once, the quantity of work corresponding to any given position in the field increases by $4\pi i$ for each time that the conductor is encircled. This quantity of work, then, is a multiple-valued function, its value depending on the number of times the conductor has been embraced by the path leading to the point considered; and the values of the function corresponding to any given point in the field differ by integral multiples of $4\pi i$.

This explains why the so-called electro-magnetic potential is a multiple-valued function, $4\pi i$ being its modulus of periodicity in the case of a single circuit traversed by a current *i*. In the fact that the work done upon a pole as it passes from one given point to another may have more than one value, lies a fundamental distinction between electro-magnetic forces and such forces, for example, as those due to gravitation. In the latter case the quantity of work [per unit of mass moved] has for each point in space one, and only one, determinate value.

Along every tube of force which embraces the axis of force (current-conductor) the magneto-motive force has the same value $4\pi i$. Now for any determinate value of the permeability the 'magnetic resistance' of a strip of the medium of given cross-section is proportional to its length, and hence for tubes of the same thickness the flux of induction is smaller in proportion as the distance from the axis is greater. On the other hand, if we divide up the field into tubes of force whose cross-sections are proportional to their distances from the axis of force, the total flux of induction will be the same for all the tubes, since the magnetic resistance of a tube varies inversely as its cross-section.

170. Energy of the field of a current loop.—In § 150 we saw that the space within a current loop, where the lines of force are most closely crowded together, is the seat of a certain quantity of energy. To evaluate this we may proceed as in § 119, where we found a measure for the energy existing throughout a magnetic field. We shall here confine our attention to the calculation of an expression for the variation of the magnetic energy around a given axis of force, *i.e.* for the energy differential dE.

In the first place it is evident that the energy of the field depends upon the current-strength i. For, when all other circumstances are given, a current-strength of 1, 2, 3 .... units produces 1, 2, 3 .... times as many lines of force per unit area at each part of the field. If, then, i is the strength of the current around the given circuit, then i will furnish a measure of the extent to which the surrounding region is filled with lines of force (compare equation (30)). If more lines of force are introduced, the current-strength remaining the same, work will have to be done, as in introducing a unit pole into a given field. This work will be greater in proportion to the closeness with which the field is already filled with lines of force—that is, it will be proportional to i. It will also be the more considerable the greater the number of new lines of force introduced into the field. Let N denote the number of lines of force originally embraced by the conductor, and dN the increment of this number, so that the work expended will be proportional to dN. If no work is lost in the process, all the work expended in increasing the number of lines of force is transformed into energy of the magnetic field; and it may be shown that

$$dE = i\,dN \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

What is here stated concerning a single current loop holds good also for any closed circuit in which a current is flowing; the increment of the energy of the surrounding field due to the introduction of dN new lines of force is always idN.

171. The field of a current as a monocyclic mechanism.— Since the field-intensity $\mathfrak{H}$ at each point of the field of a current is proportional to the current-strength i, while in the kinetic theory $\mathfrak{H}$ is proportional to the cyclic velocity, it follows that the whole distribution of cyclic motions in the field of a current has the character of a monocycle (§ 148) whose cyclic velocity q is equal to the current-strength i.

A large number of very suggestive models have been constructed (by O. J. Lodge and others) with the object of illustrating the motion of the smallest parts of the field of a current. For the most part, however, these models do not express the falling of angular velocity with increase of distance from the path of the current. Fig. 67 represents a section through the upper part of the field of a straight conductor S, the relative values of the constituent angular velocities being secured by a monocyclic connection of the moving parts (compare §§ 136–148).

BB, fig. 67, is a board 50 cm. long and 43 cm. broad, and along one of its longer sides a wooden lath 3 cm. broad is fastened to represent the current-conductor S. The further side

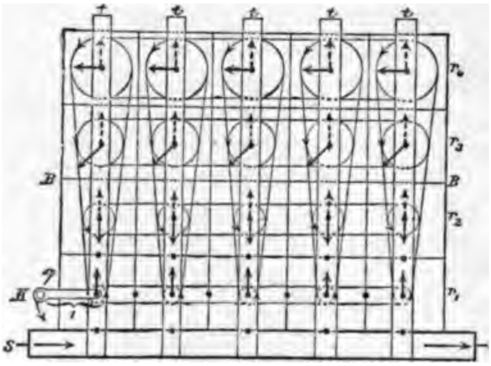

Fig. 67

of this beam (at the back of the figure) is accordingly covered with a strip of red paper of the same width, with red arrows attached to it. The remainder of the surface of the board, measuring 40 cm. × 50 cm., is pierced with four rows of five holes each, at distances of 10 cm. apart. Immediately above the five vertical series of these holes, supports t are fixed to the edge of the board BB, and serve as an attachment for five wooden laths whose lower ends are to be fastened to S, and which are pierced with holes corresponding to those in the board. Between the board and the laths are twenty grooved wooden pulleys, freely movable about axles which are in the form of short rods passing through the holes already mentioned. The pulleys in the lowest row r_1 are 2 cm. in diameter (the measurement corresponding to the middle of the depth of the groove); in the second row r_2, 4 cm.; in the third row r_3, 6 cm.; in the fourth row r_4, 8 cm. The laths having been fastened to S and to the supports t, rubber connecting bands are passed round the grooved pulleys, so that each pulley is coupled to all its neighbours. The axles of the pulleys project a little in front, and attached to them are paper discs on which arrows are marked (as shown in the figure). To the roller most to the left in the lowest row, a crank K is attached, and serves to set the system in motion. If at some given instant all the arrows are pointing vertically upward (dotted positions), and if K is turned through one complete revolution in the sense of the current S, the pulleys in the row r_4 will have turned through $\frac{1}{4}$ of a revolution, in the row r_3 through $\frac{1}{3}$, in the row r_2 through $\frac{1}{2}$, as indicated by the arrows in the figure. The angular velocity (and consequently the magnetic force which it represents) is thus inversely proportional to the distance from the current conductor. At r_1 the number of lines of force per unit area is four times as great as at r_2, and this is indicated by pegs stuck into the board, which is divided up into squares.

The motion in the entire field is determined by a single quantity, the angular velocity q of the crank K (cyclic velocity = current-strength).

C.—*Electro-magnetic flux-of-force diagrams*

Just as in Chapter V. we represented graphically the field of a magnet-pole, and constructed bipolar fields by combining two unipolar diagrams, so now we will consider how to represent the fields of rectilinear portions of a current. To this end we must agree upon a certain principle of diagrammatic representation, which, like that employed for the purely magnetic case, involves something of a conventional nature. On combining these current-fields with magnetic diagrams, we obtain representations of the action of electro-magnetic forces.

172. **Graphic representation of the system of lines of force due to a rectilinear current.**—The plane of the diagram is taken perpendicular to the current-conductor, for it is only then that the lines of force appear in their natural circular form. As before, we assume that the medium surrounding the conductor has everywhere the permeability $\mu = 1$. Thus the flux of induction, with the observation and representation of which we are really concerned, coincides with the flux of force, the lines of induction being everywhere identical with the lines of force. Moreover, out of the infinitely numerous lines of force, we shall only choose such as form the boundaries of regions through which the flux of force has the same value, namely unity, these bounding lines of the unit solenoids (§ 84) being alone represented in our diagrams. In our present inquiry we have an advantage which was lacking when we were preparing diagrams of the fields of permanent magnets, for we are able to adopt very simple conventions. In the former case, figures had to be constructed whose rotation about an axis divided the field into regions of equal flux of force, while in the case of coaxal fields we may proceed according to the following rule: Imagine a second plane to be drawn parallel to the plane of the diagram, and at a distance of 1 cm. from it. Then through every line of force represented in the diagram construct a right circular cylinder, whose axis, of course, coincides with that of the conductor. Any consecutive two of these cylinders, in combination with the two planes, enclose a canal of rectangular section, encircling the current-conductor. The lines of force represented are to be so chosen that through each such canal the flux of force is the same, and equal to one absolute unit. Since the field-intensity is inversely proportional to the distance from the conductor, the radius of the p^{th} circular line of force must

be $r_p = e^{\frac{p}{2i}}$, when that of the innermost line is 1 cm.; e being the base of the natural logarithms ($=2{\cdot}71828\ldots$) and i the constant of the coaxal system of lines of force (the so-called current-strength).

Proof of the formula.—The field-intensity at the distance r is $\mathfrak{H} = \frac{2i}{r}$, so that if we consider a narrow plane strip of surface perpendicular to the direction of the lines, of (radial) breadth $= dr$ and of height (measured parallel to the conductor) $=$ 1 cm., the number of lines of force intersecting it will be $\mathfrak{H}.\ dr.\ 1$, that is the flux of force through this element of surface will be $2i\,\frac{dr}{r}$. Hence we have for the flux of force through a plane area, whose plane passes through the axis of the conductor (and therefore intersects the lines of force perpendicularly), whose height is 1 cm. (compare fig. 68) and which lies between the radii r_0, r_1,

$$2i\int_{r_0}^{r_1} \frac{dr}{r} = 2i \log_e \frac{r_1}{r_0}$$

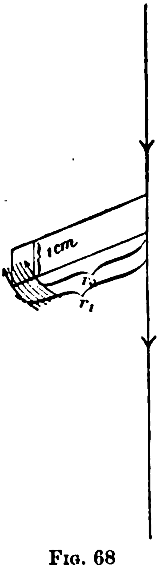

FIG. 68

If corresponding to a given value of r_0 we wish to find the value of r_1 which makes the flux of force equal to unity, we shall have to determine r_1 from the equation

$$2i \log_e \frac{r_1}{r_0} = 1, \text{ or } r_1 = r_0\, e^{\frac{1}{2i}}$$

For the next ring r_2, between which and r_1 (still with height $=$ 1 cm.) lies a surface through which the flux of force is unity, we have

$$r_2 = r_1\, e^{\frac{1}{2i}} = r_0\, e^{\frac{2}{2i}}$$

and so on; the radius of the p^{th} ring being

$$r_p = r_{p-1}\, e^{\frac{1}{2i}} = r_0\, e^{\frac{p}{2i}}$$

This result reduces to the formula given above in the special case where $r_0=1$ cm.

Fig. 69, Plate II., was constructed in this way. It represents, to a scale of one-half, the field of a long straight conductor, carrying a current of 50 ampères $=5$ deca-ampères $=5$ electro-magnetic units of current; $1/2i$ being consequently equal to $\frac{1}{10}$.

173. Deduction of the field-intensity at any point of an axially symmetrical field from the diagram.—We may use the diagram, fig. 69, as we have already done with those representing the fields of magnets, to determine the field-intensity at any given point in absolute measure. From the nature of the construction this may evidently be obtained by measuring with a rule the distance b between the two nearest lines of force on either side of the point, and dividing 1 by b. In this operation the scale to which the diagram is constructed must of course be taken into account.

That the process is justified we see at once from the relation $\mathfrak{H} \times b.1=1$. As an example we will calculate the field-intensity at a place marked × in fig. 69, and lying between the 21st and 22nd lines of force. Measuring the distance between these two lines, we find $b=4{\cdot}1$ mm. But the scale of the diagram is 1 : 2, so that in reality $b=0{\cdot}82$ cm., whence $\mathfrak{H}=1{\cdot}2$. To check this result we may make an independent calculation from the formula $\mathfrak{H}=2i/r$; the radius of the inner line of force (number 21) is in the diagram 4·1 cm., and therefore in reality 8·2 cm. For points lying on this line, since the diagram was constructed for $i=5$ deca-ampères, the field-intensity $\mathfrak{H}=1{\cdot}2$ cm.$^{-\frac{1}{2}}$ gr.$^{\frac{1}{2}}$ sec.$^{-1}$. And for points lying near to this line the field-intensity will have approximately the same value.

174. Diagram of the fundamental electro-magnetic experiment.—The changes produced by the introduction of a magnetic pole in the disposition of the lines of force in the field of a current may be graphically represented by superposing the diagrams corresponding to the two fields. But we must now represent the unipolar magnetic field in a manner somewhat different from that used in Chapter V., since we have represented the field of the current by an orthogonal plane projection of a flat region one centimetre thick, and not by a figure of rotation about an axis. We assume, then, that the effective pole of the long thin bar magnet which we introduce into the field is confined to a length of about 1 cm. at its extreme end (§ 21). This will enable us to arrive at a construction giving the variation produced by the pole in the flux of force between the two planes 1 cm. apart. The lower of these planes we suppose to be the plane of the diagram, fig. 70, Plate II., in which, for both fields, we map out the boundary lines of the strips corresponding to unit flux of force. In the case of the current these bounding lines of force are the concentric circles already referred to in § 172, on the tenth of which (reckoning zero at the innermost) the pole S is situated. Here again the scale of the diagram is 1 : 2, so that the actual distance from the current to the pole is 2·4 cm. With a current of five deca-ampères, the field-intensity at the place where the pole S is situated will accordingly be

$$\mathfrak{H} = 4{\cdot}2 \text{ cm.}^{-\frac{1}{2}} \text{ gr.}^{\frac{1}{2}} \text{ sec.}^{-1}$$

The current passes from above downwards through the plane of the paper, so that the lines of force in the field of the current pass clockwise around A, as the arrows above A indicate. For the pole, the diagram consists of a fan-shaped system of compartments, diverging from the last cm. of the length of the magnet and bounded above and below by the two parallel planes already mentioned; the space between these planes being divided up by a concurrent system of planes, which are perpendicular to them, and have equal angles between nearest neighbours. The number of compartments in the diagram of the polar field is equal to the number of units of flux of force which pass into the pole (sink) from the space between the parallel planes. We shall suppose that there are eight such units.

The strength of the pole is thus greater than 8, since lines of force also pass to it from above and below.

In the plane of the diagram the eight partition-planes appear in orthogonal projection as eight straight lines, passing through S and ranged round at equal angular intervals. That line of force of the current-field on which S itself is situated passes without deviation directly into this sink. A line of force belonging to S must therefore be tangential to it, and thus we arrive at the relative disposition of the two diagrams shown in fig. 70.

On either side of this tangential line, four units of flux of force pass from the surrounding field into the pole, where they appear to end, disappearing from the field, and passing into the magnet. The direction of the lines of force at each point is to be found by combining the two diagrams together. The resultant course of the bounding lines of force is obtained by drawing the proper diagonals across each of the four-sided figures which are formed by the crossing of pairs of neighbouring lines of the two systems—those diagonals, namely, which start from the same corner of the quadrilateral as do the lines of force. At the sink S itself, the resultant lines are all tangential to the straight lines of the polar system. Fig. 70, which was constructed in this way, should be compared with the iron-filing diagram fig. 57, which corresponds approximately with the lower of the two parallel planes here considered, except that the current and its lines of force are reversed in direction.

Eight of the flux of force channels surrounding the current are collected together at S like the tail of a comet; and the lines of force, which were originally in the form of closed curves, have become opened out into infinite spirals.

If the south pole S were free to follow the pull exerted by the lines of force, it would tend to move round the current-conductor A in the counter-clockwise direction, since it strives to move along the lines of force which terminate upon it. It might thus appear that a rotation of the pole around the conductor could be maintained indefinitely by electro-magnetic forces, thus providing us with a veritable 'perpetuum mobile.' It must not be forgotten, however, that we have here assumed the fulfilment of impossible conditions, since a free isolated magnetic pole cannot exist, not to mention that energy must be continuously expended to maintain the magnetic field of the current, as we shall see later.

CHAPTER X

COILS, SOLENOIDS AND ELECTRO-MAGNETS

Having now become acquainted with the simple current, its lines of force, and their action on magnetic systems, we proceed to consider some forms of current-conductors which are of great theoretical interest as well as of practical importance. They are all developed from the concentric axis of force system in such a way as to collect together a great number of lines of force through some definite portion of the field of the current. There are two ways of arriving at this result; in the first place, we may collect into a small compass some considerable length of the conductor itself, by winding it in the form of a coil or bobbin; and in the second place, by the introduction of masses of iron we may greatly increase the permeability of the medium through which the lines of force have to pass. By the latter method we may convert a coiled current-conductor into an electro-magnet, of far greater strength than any natural magnet, or even than any of the compound permanent magnets, which we have been so far able to prepare. Such electro-magnets enable us to study the magnetic properties of various substances much more accurately than would be possible with weaker magnets of steel.

175. **Coils.**—We have already (§ 149) described how the lines of force in the field of a current become collected together into a bundle, when the conductor is bent into the form of a loop. We may further increase the number of lines of force by adding more turns of wire to the loop. We shall cause the more lines of force to thread through the loop, the greater the length of the conducting wire we wind on, *i.e.* the

greater the number of turns of wire; a conductor of this kind constitutes one form of 'coil.' The straight line through the centre of the windings, perpendicular to their plane, is called the axis of the coil.

The sum of all the plane areas bounded by the separate turns of wire is called the total area of the windings, a quantity which is of importance in determining the number of lines of force which thread through the coil. When the current is measured in ampères, we speak also of the number of 'ampère-turns,' meaning the product of the current-strength and the number of turns. When the diameter of a (ring-shaped) coil is given, the field-intensity is proportional to the number of ampère-turns.

Fig. 71 shows the line-of-force diagram for a ring-shaped coil, in a plane passing through the axis of symmetry.

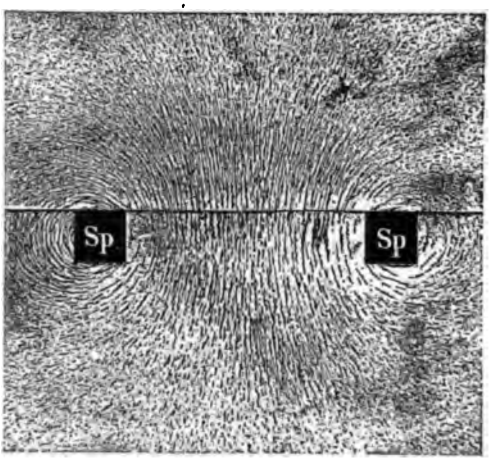

FIG. 71

Sp, *Sp* are the places where the coil intersects the plane of the diagram, on either side of which it projects semicircularly. The lines of force surrounding all the separate turns follow approximately the same course, and unite to form continuous curves, those which most closely embrace the cross-section of the coil being nearly concentric with it. On the other hand the more remote lines, owing to the pressure of the inner ones, are bowed out excentrically into widely spreading curves. In the neighbourhood of the axis of the coil, the field is approximately uniform.

The line-of-force diagram, fig. 71, was obtained in the following way. Upon a ring-shaped wooden frame, 1·7 cm. broad, 2·0 cm. deep, and 11 cm. internal diameter, many turns are wound of covered copper wire 1 mm. in diameter, and the frame is supported vertically upon a stand. A piece of thick cardboard, with a rectangular hole cut in it, is slipped over the ring and arranged so that its upper face is horizontal, and passes exactly through the centre of the ring. Upon the cardboard is laid a sheet of paper, with rectangular notches cut in one edge, to receive the cross-sections *Sp* of the coil, another sheet of paper with a straight edge being laid with this edge in contact with the notched edge of the former sheet. Thus we obtain even the central portions of the diagram, and by separately removing the two sheets from their positions and joining them together again, we are enabled to photograph the diagram without any obstruction from the upper part of the coil. Where the two sheets of paper meet, a dark line appears in the figure, running just above *Sp Sp*.

176. **Multipliers.**—These are devices for detecting and measuring even feeble currents, and depend upon the principle of collecting together as many as possible of the lines of force due to the current. To produce more considerable effects upon a magnetic needle than would be possible with a simple current loop, the lines of force are augmented in number, by superposing many turns of wire. It was in this way that the first sensitive instruments for detecting and measuring currents were constructed (SCHWEIGGER and POGGENDORFF, 1820).

The effect may be still further increased by using, instead of a simple needle at the centre of the coil, an astatic pair of needles, one of which lies within the coil while the other, oppositely magnetised, lies above them. This arrangement of needles diminishes the influence of the earth's magnetism, which tends to restore the suspended system to its position of rest (§ 45), while it adds together the effects produced by the windings upon the inner and outer needles, as may be easily verified by means of the 'thumb rule' (§ 160); the right hand, pointing in the sense of the current, being laid upon the upper part of the windings, first with the palm turned towards the inner needle, then towards the outer.

Since those ends of the needles which point in either one direction are of opposite polarity, the effects produced upon them by the field of the current will reinforce one another.

177. **Galvanometer.**—The measurement of a current by deflection of a magnetic needle may be made more accurate when, instead of a single coil of wire, two coils side by side are used, the lines of force proceeding from the one being immediately taken up and kept together by the other. In this way we avoid a divergence of the lines of force at the place where their effect is to be compared with that of the earth's magnetism, the field between the two coils being nearly uniform. Here the magnet hangs, suspended by a cocoon fibre or thin quartz fibre; it may conveniently be in the form of a thin circular disc made of steel, and so magnetised that when it is suspended its magnetic axis is horizontal. If one face of the disc is polished, the deflections of the reflecting magnet may be very accurately read through the aperture of the coil by means of a scale and telescope (G. WIEDEMANN's *mirror galvanometer*). In place of the steel mirror, SIEMENS introduced a bell magnet, to whose stem a mirror was attached. For reasons to be explained later, the steel disc or bell magnet is surrounded by a mass of copper, and the suspended system is protected from draughts by a glass case.

More recently systems of several needles have been employed, arranged between two pairs of coils. A light silvered glass mirror is attached to the middle of a flattened aluminium wire. Above and below the mirror, on both sides of the wire, short slender pieces of magnetised watch-spring are fastened, all the magnets belonging to the upper set having their north poles turned in one direction, while all those in the lower set have their north poles turned in the opposite direction. If there are the same number of magnets above and below the mirror, and if all the individual magnets are about equally strongly magnetised, the whole system is very nearly astatic, the effect of terrestrial magnetism on the upper system being almost exactly neutralised by its opposite effect on the lower system. On the other hand, the magnetic moment of either system is relatively large, if we take account of its lightness. Each system of needles lies between a pair of coils, one pair being traversed by the current in one sense, and the other pair being traversed in the opposite sense. If the suspension is made by means of a thin quartz fibre, these instruments may attain to great sensitiveness, detecting even the billionth of an ampère. (Lord Kelvin, H. E. J. G. du Bois and Rubens.)

178. **Solenoids.**—Let a long stiff wire be wound in the form of a helix, a certain space being left between consecutive turns, so that we may easily see what goes on in the interior. The wire tube thus formed is called a 'solenoid,' and must not be confused with the 'unit solenoids' spoken of in § 84, the two uses of the term being entirely distinct. We will investigate the disposition of the lines of force in the neighbourhood of such a coil when it is made the axis of a concentric distribution of force.

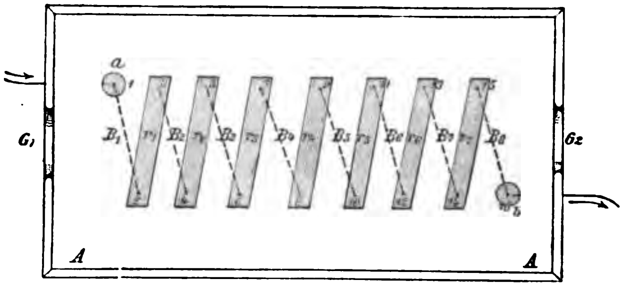

Fig. 72

In a board AA, fig. 72, 12 cm. by 25 cm. and furnished with a projecting rim, two small hollows a, b are sunk, and are filled with mercury, into which wires from without are made to dip. A series of oblique parallel grooves r_1 to r_7 are also cut in the board and filled with mercury. Upon the board AA is laid another of equal size, which is kept in position by resting upon the projecting rim of the former; two semicircular openings being cut in this rim to allow the upper board to be conveniently lifted off. The upper board is pierced perpendicularly with a series of holes (numbered from 1 to 16), and through these are thrust wire hoops B_1 . . . . B_8. The free ends of these hoops are straight, and amalgamated at the tips, which dip into the mercury troughs (or cups) in the manner shown in the figure. Thus the hoops represent the upper portions of eight turns of a solenoid, the mercury troughs representing the lower portions. The phenomena will be very little influenced by the fact that the lower portions of the windings are not curved like the upper.

Upon the upper board is laid a sheet of paper pierced with corresponding holes. Over this fine iron dust is evenly scattered, and then the wire hoops B are arranged in position, so that they dip into the mercury below and establish a continuous metallic connection between *a* and *b*. After the current has been stopped, the wire hoops may be carefully lifted out again, and the line-of-force diagram rendered permanent, either by means of a spray of one of the solutions mentioned in § 7, or by transferring the iron particles to gummed paper, or by photography. It was by this last method that fig. 73 was prepared.

Line-of-force diagram in a plane through the axis of a solenoid (fig. 73).—Each of the black dots in the figure represents a cross-section of the conductor, and may be recognised as such by the encircling lines of force which are collected about it. But since the current has the same direction in adjacent windings, the more remote lines of force merge into one another. Thus within the solenoid there is a continuous flow of lines of force which are bent into a sinuous form in the immediate neighbourhood of the current-conductor. Each turn of the solenoid encompasses fresh lines of force and carries forward the flux of force

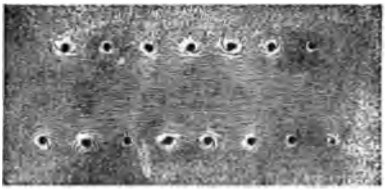

FIG. 73

from the preceding turn. Thus within the solenoid the magnetic force is very strong, while outside there are hardly any lines of force. The more nearly we approach the axis of the solenoid, the more exact is the parallelism between the lines of force; the field is uniform. Fig. 78 further illustrates the cross-pressure which the lines of force exert upon one another. The outer lines of the system bulge out into the spaces between the turns of wire, like elastic threads tied up tightly into a bundle, while just under the wire they are closely crowded together. Such a solenoid has far greater field-energy than its separate windings. At its ends, the lines of force diverge in all directions.

179. Equivalence of solenoids and bar magnets.—At one end of a solenoid lines of force emerge as they do from the north pole of a permanent magnet, while at the other end they re-enter the solenoid, as would be the case at the south pole. ***Thus a solenoid, so far as concerns the external disposition of its lines of force, behaves exactly like a bar magnet.***

If we follow out the direction of the current, we have at once a sufficient datum for determining from which end of the solenoid the lines of force emerge, which end, that is, corresponds to the north pole of a magnet, as well as the end where the lines re-enter, that is, the magnetic south end. Since the current-conductor embraces the lines of force in the clockwise sense, these lines will be passing into the interior of the solenoid at that end which, viewed from the outside, corresponds to a clockwise circulation of the current. On the other hand, that end which corresponds to a counter-clockwise circulation must appear as a source of lines of force, that is, as a north magnetic pole.

Model of the solenoid with its lines of force.—A helix, 30 cm. long and 5 cm. in diameter, is wound from thick copper wire overwound with red cotton or silk, the direction of the current in the wire being indicated by means of red arrows. Through the helix are threaded a number of model tubes of force (fig. 39), which are bent round into the form of closed curves (§ 127), their arrangement being such that the rotatory motions corresponding to the magnetic forces follow the same direction as the current.

These model solenoids are to be preferred to the usual cylinder with arrows marked upon it.

So far as concerns the external disposition of lines of force, our model will correspond exactly to that of the tubular magnet in fig. 42.

We may here notice how the model, with its interior rotational motions, may be used to explain the phenomenon of the electro-magnetic rotation of the plane of polarisation of light. If we suppose the periodically alternating circumstances which constitute a beam of light to be propagated along the lines of force through the interior of the solenoid, the variations of conditions, which take place transversely to the lines of force, will always be communicated to particles which are themselves in rotation about the direction of the lines of force as axis. We might be led to expect, then, that an alternation of conditions originally taking place in some determinate direction (a so-called plane-polarised beam of light), on being propagated along the axis of a helical current, would be somewhat rotated in the direction of the latter. This consideration formed the basis of Lord KELVIN's and MAXWELL's conclusion that rotational motions exist in a magnetic field.

The following experiments show that a solenoid does actually behave like a magnet.

Experiment 58.—A long open wire helix, with arrows indicating the direction of the current and the corresponding direction of the lines of force within (most conveniently the model just described), is connected to terminals by means of conducting wires, so that it is traversed by a current. It is then brought near to the poles of a declination needle. All the deflection experiments described in § 38, corresponding to both the principal positions, may now be performed with the solenoid, just as they were previously performed with the bar magnet.

Experiment 59.—To the ends of a solenoid made of thick aluminium wire, gold threads are attached by which it is hung up. When a permanent magnet is brought near to the solenoid on one side or the other, corresponding deflections take place, and when the magnet is axially approached to the solenoid, phenomena of attraction and repulsion are observed, like those described in § 33.

Experiments may also be performed with **Ampère**'s suspension (§ 151), the solenoid being provided with points which dip into mercury cups, but this method is not so convenient as that of the aluminium wire helix suspended by gold threads.

Although there is so close a resemblance between solenoids and magnets of the same external form, there is yet a very essential difference between them; this arises from the different spacial relations (cyclôsis) existing in the fields of magnets and of currents. In the case of steel magnets, we cannot directly follow the course of the lines of force in the interior, we only suppose the lines are taken up and transmitted by chains of molecular magnets.

In the case of the solenoid, the course of the lines of force may be followed without hindrance throughout the whole interior, the medium being the same as in the surrounding space, and having the same permeability; we may pass along the lines of force right through the helix from one side to the other, without encountering surfaces which separate media having entirely different magnetic properties.

180. **Bobbins.**—The effects produced by a solenoid are greatly increased when a greater number of lines of force are made to thread through its interior, that is, when greater and greater lengths of wire are wound upon the helix. When the length of the helix is increased the 'end effects' become less important, that is to say, the irregularity of distribution of the lines of force always existing at the ends, which behave in regard to the external field as sources of lines of force (compare fig. 73).

Solenoids of this kind, closely wound with several layers of insulated wire, we shall refer to as bobbins.

They must necessarily give rise to the same phenomena as the solenoids already examined, only in a greater degree, corresponding to the increased number of the lines of force.

Thus a bobbin is also characterised by the emanation and divergence of lines of force from one end (N), and their convergence to the other end (S), where they re-enter the helix.

Fig. 74 shows a view from above of an arrangement suitable for producing the fields with which we shall now be concerned. AA is a base-board, 16 cm. × 40 cm., standing on feet and provided with a wooden rim. From it project two supports BB, 18 cm. apart, each having a circular hole, 2·5 cm. in diameter, bored through its upper half at the same height above AA. Through these holes is fitted a pasteboard tube C, of the same (external) diameter, and with walls about 2 mm. thick. In the figure, C is indicated by dotted lines. Insulated copper wire 1 mm. thick is wound upon it. The apparatus used for producing figs. 75–78 had five layers of windings, each consisting of 100 turns of wire, so that the total number of turns was 500, and the number of turns per unit length of the helix was 28. The ends of the wire are brought to FF.

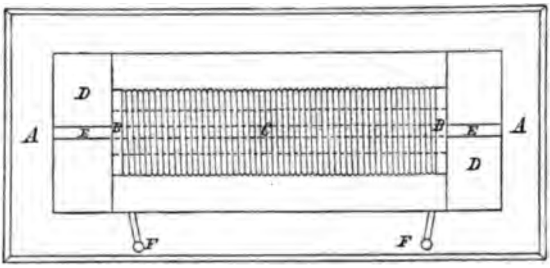

FIG. 74

To enable the observer to investigate the disposition of the lines of force inside and outside the helix, and especially at the ends, the lower half of the pasteboard tube is filled by a semi-cylinder of wood; wooden shelves DD are also fixed to the supports BB, at such a height that their upper surfaces lie flush with that of the wooden semi-cylinder. At the same height again there are wooden shelves running alongside the bobbin and terminating where they meet DD. Upon the horizontal table thus constructed is laid a sheet of paper, which has notches of the appropriate form cut in it, so that it may project far into the interior of the bobbin, and at the same time may fit closely round the outer boundary of the windings.

It is especially important to render directly evident the behaviour of permanent magnets and of pieces of soft iron in the field of a solenoid. To allow this, a groove EE is cut in the shelves DD and the inner wooden semi-cylinder, so as to run right through the helix from side to side. In this groove square-prismatic bar magnets or bars of soft iron may be laid.

181. **Magnetic effects of bobbins.**—We proceed to consider a number of experiments with helices traversed by currents, showing the disposition of the lines of force in arrangements which have important applications, especially in technology.

Field of the pole of a bobbin.—We shall first consider the undisturbed divergence of lines of force from the end of a bobbin (fig. 75). The cross-sections of the windings appear as two black patches in the lower part of the figure. The turns of wire, it must be remembered, do not extend right to the ends of these cross-sections, since the thickness of the support B intervenes (fig. 74). Within the bobbin the field is uniform, but where the lines of force appear, to the left of the diagram, they are already beginning to diverge. After the black rectangular patches which represent the section of the helix, there are no more windings, and here accordingly the lines of force diverge regularly and without hindrance into the external field. It will be noticed how the strong stream of lines of force through the interior of the bobbin spreads out on all sides as it passes through the open end. In the field facing the bobbin we shall place first of all bodies which are themselves sources of lines of force, and in the next place such bodies as have a far higher permeability than the surrounding medium.

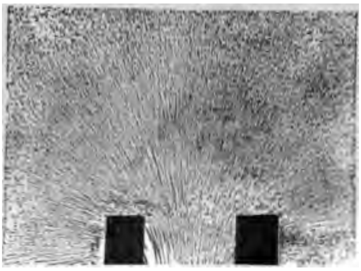

Fig. 75

(*a*) *Action of a bobbin upon a permanent magnet.*—In the channel E of the apparatus, fig. 74, is laid a powerful bar magnet of rectangular cross-section, the end of the magnet and the end of the bobbin which are turned towards one another being of the same polarity. Fig. 76 shows how the lines of force issuing from the interior of the bobbin W W are deflected on encountering the lines which emerge from N, bending round this pole as they might round an impenetrable obstacle. Like the stream lines from two neighbouring sources which send forth fluid against one another, the lines of force emanating from the helix produce a bending in the lines which proceed from N, and which ultimately pass round to the south pole (outside the figure), beyond the indifferent zone *i* of the magnet. Thus it is that there is an indifferent point or zero point J between the end of the bobbin and the pole of the magnet, the field-intensity being inappreciable through a small region surrounding this point. If the magnet is movable, it will be driven away axially from the pole of the bobbin, owing to the pressure across the direction of the lines of force. On the other hand, when the magnet is fixed, the bobbin, if movable, will be thrust in the opposite direction, that is, away from the magnet.

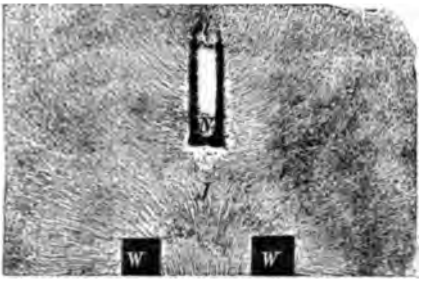

Fig. 76

If the magnet pole of opposite name be turned towards the end W W of the helix, the phenomena will be reversed, the line-of-force diagram being similar to fig. 77. The lines of force emerging from the bobbin stream into the south pole (sink) of the magnet. The tension along the lines of force tends to drag the magnet into the interior of the bobbin, or, when the magnet is held fixed, to drag the bobbin over it like an enclosing case.

The mechanical actions of bobbins upon coaxally placed bar magnets were made the basis of many of the older electro-motors. When the current through the bobbin is made to alternate in direction, the magnet is correspondingly drawn in and thrust out, so that a reciprocating motion results, like that of the piston of a steam engine.

(*b*) *Action of a bobbin on soft iron.*—Since iron collects together the lines of force, that portion of the iron bar which is turned towards the bobbin becomes a sink (south pole); hence the iron is drawn towards those places where the lines of force are densest, that is, from the external field into the interior of the helix, and this independently of the direction of the current. Herein lies the wide applicability of the phenomenon, for example, in the regulation of arc lamps.

In the channel E of the apparatus, fig. 74, is laid a short piece of soft iron. If now a current be made to circulate in the bobbin in either direction, the line-of-force diagram obtained will be like fig. 77. The lines of force emerging from the interior of the helix, between the windings W W, do not diverge equally in all directions as in fig. 75, but pass for the most part to the soft iron F*e* of greater permeability, entering it at *e* and making their way through its length. At the further end *a* they once more emerge into the field. In the neighbourhood of *a* the lines of force are much denser than if no iron were present. The end *e* has become a 'sink,' and the end *a* a new source.

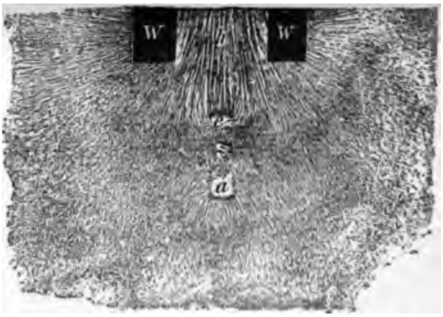

FIG. 77

The remaining lines of force, which have not passed through the iron, are spread out less widely than they would be in the undisturbed field. The iron has collected together the lines of force to such an extent that the field from *h* to *e* in the figure is nearly uniform. If the iron rod F*e* is free to move, it will be drawn into the interior of the helix owing to the tension along the lines of force which terminate at *e*.

The strength of the pull exerted upon iron depends upon the number of lines of force concerned. This number is approximately proportional to the current-strength, for the measurement of which the phenomenon may accordingly be employed.

Experiment 60.—A helix wound with many turns of wire is supported vertically upon three feet, and an iron core which can pass easily through the helix is supported below with its upper end just within the windings. When a strong current is passed through the helix, the iron core is lifted up and sucked into the interior of the coil; on breaking the circuit it falls down again.

182. **Spring ampère-meter.**—A bobbin of wire is fastened in a vertical position upon a stand, or against a wall, and a long vertical guiding rod is fixed so as to be coaxal with the bobbin. Sliding upon this rod within the bobbin is a thin cylindrically bent piece of sheet iron, which is attached to a helical spring above. When there is no current flowing, only the lower end of the sheet-iron cylinder dips into the interior of the bobbin, but when the circuit of the current is closed, the cylinder is drawn down into the bobbin against the resistance of the spring, the displacement being the greater the stronger the current employed. The value of the current in ampères is given by an index attached to the iron cylinder and moving over a vertical scale; this latter having previously been empirically graduated so as to give 'direct readings' (F. KOHLRAUSCH).

In lectures and demonstrations it is very convenient always to be able to show the value in ampères of the currents used in an experiment; in this way not only is the meaning of the term kept in view, but a definite conception becomes associated with a definite number of these practical units. It is therefore very desirable to be able to show to a large audience a measuring instrument of considerable size and of a type commonly employed, especially in technology. KOHLRAUSCH'S spring ampère-meter is convenient for the purpose, as without much expense it can be made of such dimensions that its indications can be read from a considerable distance.

183. Uniform field within a helix.—A bobbin traversed by a current provides one of the most important ways of producing a strong uniform magnetic field. If the bobbin is sufficiently long, the effects of the free ends soon become negligible as we pass to the interior, where the lines of force run almost exactly parallel to one another. We shall make use of the uniform field in the interior of a helix to render evident the high permeability which is characteristic of soft iron.

Once more place the short iron bar in the channel E of the apparatus, fig. 74, and fasten to it with sealing-wax a strip of cartridge paper whose breadth is equal to the bore of the bobbin. After sprinkling the paper uniformly with iron dust, push it, along with the iron bar, into the interior of bobbin. On closing the circuit of the current for a short time, and tapping the apparatus, we obtain a line-of-force diagram such as fig. 78.

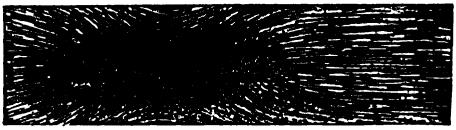

FIG. 78

The lines of force run parallel to one another in the interior of the helix, until they approach the extremity of the iron rod Fe, towards which they strongly converge. They pass through the iron, and emerge again at the further end, where they once more spread out; but at greater distances from the iron they become more and more parallel to one another, so that the field regains its uniform character. Lines of force crowd into the iron from the surrounding field. *Soft iron in the interior of a helix traversed by a current collects together the lines of force.*

Since the lines of force which enter Fe at one end are exactly the same in number as those which leave it at the other end, and since the lines of force at a little distance neither converge nor diverge, the mechanical forces exerted upon the iron will just neutralise one another's effect, so that there is on the whole no force tending to move the iron in one direction or the other.

The directing influence exerted upon iron particles by the uniform field in the interior of a helix may be rendered evident to a large audience by projection with a lantern as follows: A short wide glass cylinder is provided with two glass discs which can be screwed firmly against its ends so as to close them (as in apparatus for investigating the rotation the plane of polarisation by a sugar solution), and upon this cylinder several layers of thick insulated copper wire are wound. The cylinder is filled with a mixture of glycerine and iron filings, and is placed in front of the lantern, in the axis of the emergent beam of light. The picture of the cross-section of the cylinder, when formed upon the screen, appears dark, because the light is cut off by the irregularly distributed iron filings. If, however, a strong current is sent through the helix, the particles arrange themselves in axial chains, and allow light to pass.

184. **Magnetising helix.**—The strong uniform magnetic field which exists in the interior of a helix through which a current is flowing is used to magnetise steel bars. This is a far more effective method of magnetisation than that of rubbing with another magnet, as described in § 19, for in the present case we make use of far denser bundles of lines of force, and can therefore cause a much greater number of lines to pass through the steel, even when the coercive force (§ 18) is greater. If the helix is longer than the steel rod to be magnetised, the magnetic force due to the current is everywhere the same along the length of the rod, which thus becomes very uniformly magnetised.

Experiment 61.—Let unmagnetised steel knitting-needles be placed within a long helix through which a current is flowing. On stopping the current and removing the needles, the latter will be found to act as bar magnets of considerable strength.

It has been already mentioned that the number of lines of force which thread through the interior of a helix depends on the number of turns of wire, and on the strength of the current, that is to say, on the number of 'ampère-turns' (§ 175). Within a long helix the field-intensity is the same for all cross-sections which are sufficiently far from the ends, its value being immediately deducible from the number of ampère-turns per centimetre of length.

If l is the length of the helix, and if there are n turns of wire in all, then n/l is the number of turns per unit of length. If, then, there is a current of i ampères flowing in the circuit, the number of ampère-turns per cm. length of the helix will be $n/l \times i$. This must be proportional to the field-intensity at any point within the helix. From considerations similar to those adduced in § 165 we may see that the field-intensity $\mathfrak{H}$ corresponding to a current of i ampères through n turns of wire in a helix l centimetres long is

$$\mathfrak{H}=4\pi\frac{n}{l}\frac{i}{10} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

In the bobbin, fig. 74, n/l was about 28, and the current employed in making the line-of-force figures had a strength of 20 ampères = 2 deca-ampères (*i.e.* 2 absolute electro-magnetic units), though such a current could only be kept circulating through the coil for a very short time. Thus the field-intensity in the interior was about 700 units, so that a unit pole placed within the helix would experience a force of 700 dynes, which, roughly speaking, is equal to the weight of 7 grams. In other words, 700 lines of force pass through each square centimetre of the cross-section of the interior. In the earth's magnetic field we found that there was about one line of force to each 2 cm.^{2} of perpendicular surface, so that our field is approximately 1400 times as strong as the terrestrial magnetic field in Central Europe.

185. **The electro-magnet.**—The magnetic permeability of soft iron is more than 1,000 times as great as that of air. If, therefore, the interior of a helix is filled with this material, there will be a great reduction of 'magnetic resistance,' so that corresponding to a given number of ampère-turns

per unit length (that is, to a given magneto-motive force) we shall have a far greater number of lines of induction threading through the helix. At either end of a strong helix provided with an iron core the number of lines of force entering or leaving must accordingly be very great. A helix thus provided with an iron core is called an *electro-magnet.*

Experiment 62.—A bobbin has the ends of its wire joined to flexible current-conductors, and is brought near to some iron nails; several of these will be drawn into the interior of the bobbin or will be held in contact with the ends. If now we introduce into the bobbin an iron core which exactly fills it, the magnetic effects will be enormously increased, though an ampère-meter placed in the circuit shows that the strength of the current is unaltered. If the circuit is now broken (at some distant point) the nails fall away from the iron core, the magnetic effects having ceased, except for a small amount of 'residual magnetism.'

Since the iron core only increases the permeability of one part of the field of the current, the polarity of the ends of the helix (§ 179) will remain unchanged:

The north pole of an electro-magnet is that which (viewed externally) is encompassed counter-clockwise by the current, the south pole being that which is encompassed clockwise.

If a complete iron ring is overwound with one or more layers of insulated wire, through which a current is made to circulate, we have thus an electro-magnetically excited toroid. Here again the magnetic force in the interior is related to the number of ampère-turns per unit of length in the manner given by equation (33), § 184. Owing to the small 'magnetic resistance' of the circuit, and the absence of all demagnetising end-effects (§ 96), a toroid of this kind may acquire a very high degree of magnetisation $\mathfrak{J}$, the values of the flux of induction I being correspondingly great. Accordingly such endless helical forms and closed electro-magnetic circuits are largely used in investigating the magnetic properties of different kinds of iron, etc.

186. **Some applications of electro-magnets.**—The applications of electro-magnets are so numerous and varied that we can here attempt no more than a very slight account of them. In the first place, all the experiments with permanent magnets described in Chapters I. and II. may be repeated with electro-magnets with far more powerful effects. To this end the electro-magnets are to be joined to 'terminals' by means of flexible conductors, or supported by strong gold threads.

Iron bars, either straight or bent into a horse-shoe form, may be overwound with insulated wire, thus furnishing us with electro-magnets of the bar or horse-shoe type. The fields of these have the same properties as those of the corresponding forms of permanent magnets. To show the form of their fields the coarsest iron filings may be used; these stand out to a considerable distance from the electro-magnet, and render evident peculiarities in the remoter parts of the lines of force, such as could not be recognised in the case of ordinary magnets.

The lifting power of electro-magnets is very great, especially of those of the horse-shoe form. The iron armature sometimes remains attached to these latter even after the current has ceased to flow, an effect which is due to residual magnetism.

This residual magnetism in the cores of electro-magnets plays an important part in the working of the dynamo, as we shall see later. The poles of a large electro-magnet, of which we shall repeatedly have to make use, are distinguished by coverings of red and blue paper respectively.

Experiment 63.—A little bar of bismuth is hung up by a silk fibre attached to its middle, so that its axis rests horizontally, and a bar of iron of the same dimensions is suspended in a precisely similar manner. If the bismuth is brought between the poles of a powerful electro-magnet, it sets itself across the direction of the lines of force, in virtue of its diamagnetic properties (equatorial setting, § 105). A much weaker magnetic field will suffice to show that the iron bar sets itself along the direction of the lines of force.

But the great applicability of electro-magnets depends not so much on their strength as on the rapidity and ease with which their magnetic properties can be made and destroyed, especially since these changes can be effected from a distant position. All that need be done is to make or break contact at any point of the galvanic circuit. This is sometimes effected by the electro-magnet itself, as in the case of automatic *contact-breakers* (WAGNER's hammer), continuously excited tuning-forks, and electric bells. The armature attracted by the electro-magnet is of soft iron, and is fastened to a spring which presses against a platinum point on the side remote from the magnet. The current passes between the platinum point and the spring, and through the fixed support to which the further end of the spring is attached, after which it passes in a helical path around the iron of the electro-magnet. When the circuit of the current is closed, the electro-magnet sends forth lines of force which pass very largely through the armature. This is accordingly attracted towards the magnet so that it no longer makes contact with the platinum point: the circuit is broken, the electro-magnet loses its magnetism, and the armature springs back to its former position. But as soon as the spring presses on the platinum point, the circuit is once more completed; thus the cycle of operations commences afresh, and continues to repeat itself automatically without any need for external interference.

In telegraphy, the circuit of a current is closed by depressing a key at the one station; this causes lines of force to embrace the conducting telegraph wire, a large number of these lines being gathered together by the core of an electro-magnet which is set up at the other station, and whose coils are included in the circuit. The electro-magnet thus attracts an armature which is attached to a mechanism of one kind or another, so that either marks are made upon a strip of paper drawn through the instrument—a dash for a continued contact and a dot for a momentary contact—

as in Morse's writing telegraph, or else some definite symbol upon a type-wheel is brought into position and printed upon the paper, as in Hughes's printing telegraph.

Electrical clocks depend upon the making and breaking of an electrical contact by the pendulum of a standard clock. A metallic point is attached to the lower end of the pendulum, and dips for a moment during each swing into a drop of mercury, thus closing a circuit which includes a series of electro-magnets. These accordingly become magnetised and, each attracting an armature, cause a wheel in a system of clockwork to turn through the space of one tooth. In this way a single clock at a central station may be made to indicate the time with certainty at a number of secondary stations.

CHAPTER XI

FORCES EXERTED UPON A CURRENT-CONDUCTOR IN A MAGNETIC FIELD ('LEFT-HAND RULE')

In Chapter IX. we gave some account of the mutual actions of currents and magnets, the former being supposed fixed while the latter were free to move. But these mutual actions give rise to a new series of phenomena when long or short portions of a current-conductor have freedom to move through a magnetic field. Here again one of the fields concerned in the action has a coaxal distribution, and it is therefore not surprising that in many cases the forces exerted are such as produce rotations.

187. **Movable sliding-piece in a uniform field.**—In order to determine the force exerted upon a current-conductor in a magnetic field, a portion of the conductor must be given such freedom of motion as will allow it to follow in the direction of the impressed force, without at the same time severing its connection with the fixed parts of the conductor. In nearly all the arrangements designed for this purpose there is a sliding-piece in the circuit of the conductor—that is, a rod or wire sliding upon a pair of rails or channels filled with mercury, each rail or channel having one end connected to wires which form part of the circuit. The blind ends of the rails or channels which project beyond the slider are not traversed by the current, and give rise to no lines of force.

Fig. 79 shows a sliding arrangement of this kind, suitable for exhibiting the phenomenon to a large audience. The base-board AA, 45 cm. long and 20 cm. wide, is provided with feet, with a projecting rim and with a hole which can be closed or opened at pleasure; this last being intended to facilitate the removal of any mercury which may have been spilt. To AA are fastened two vertical stems BB, which in turn support the board CC. Suspended by the two threads $f_1 f_2$ is a hoop of copper wire *m*, bent twice at right angles, and loaded with a small piece of brass; this constitutes the sliding-piece. The ends of *m*, which are bent downwards, are armed with narrow blades of platinum foil (not specially indicated in the figure, where they appear foreshortened to straight lines). These blades dip into mercury contained in channels which are excavated in wooden bars H_1, H_2, seen in end view in the figure. The bars H_1, H_2 are provided with projecting pins, which fit into holes in the board AA and serve to keep the bars fixed in their proper places. From them thick but easily removable metallic connections (conducting wires) pass to the terminals K_1, K_2, these connections being kept above the base-board, so as to be easily recognised. Between the two wooden bars is a block of iron E, which serves to collect together the lines of force.

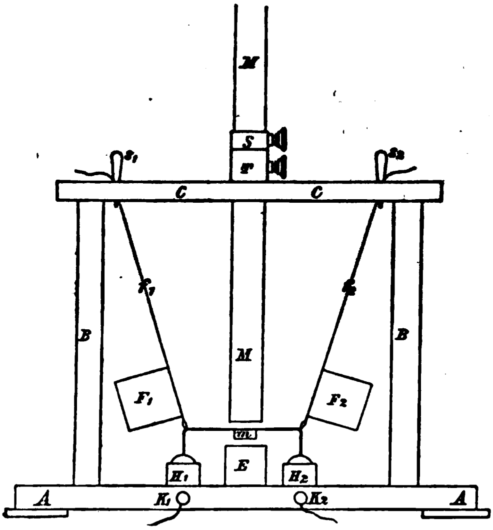

FIG. 79

The threads f_1, f_2 are attached to the board CC by means of the pegs s_1, s_2. The paper vanes F_1, F_2 allow even small to-and-fro movements of the wire hoop to be seen from a distance, especially when the narrower side of the apparatus is turned towards the audience. Above the movable wire and within a little distance of it is the strong bar magnet MM, which can be clamped in position in the brass socket T, attached to the board CC. When the circuit of the current is closed, the hoop *m* is displaced to one side or the other in the magnetic field between M and E, and when the circuit is broken again the hoop returns to its position of rest. If it is desired to maintain the field of the current continuously, and to move the magnet MM nearer to and further from the movable wire, the brass sleeve S may be firmly clamped upon the magnet, while the screw belonging to T is relaxed, so that now the magnet may be rapidly lowered to within a little distance of the hoop *m* without fear of an actual contact between the two.

When a magnet-pole is approached to a movable wire conveying a current, or when a current commences to flow in a movable conductor placed in a fixed magnetic field, the forces acting upon the conductor in question are found to be perpendicular to the lines of force of the magnetic field, and also perpendicular to the conductor itself. Thus with the view of our apparatus shown in fig. 79, the displacement of the hoop *m* is towards or from the observer. Eight permutations of the experimental conditions may be made: (1) Magnet fixed (a) north pole downward: (α) current in one sense (β) current in the opposite sense (superposition of a current-field upon a magnetic field already existing). (b) south pole downward: the displacements corresponding to (α) and (β) are now reversed in direction. (2) The current flowing continuously, the magnet is lowered towards the movable wire (a) with north pole downward (b) with south pole downward. When the direction of the current is reversed (α) and (β) give corresponding reversal of the direction of displacement (superposition of a magnetic field on an already existing current-field, which is driven away by it).

The displacement of a movable conductor in a magnetic field thus exhibits a twofold symmetry, a reversal of direction being produced by a reversal of the current as well as by a reversal of the lines of force in the field of the magnet. The direction in question, then, depends both on the direction of the current and on that of the lines of force of the magnet.

We shall use blue arrows to indicate the direction in which the conductor moves, just as we have already used red arrows to indicate the direction of the current in the conductor, and white arrows for the direction of the lines of force.

Experiment 64.—The same phenomena may be shown with less completeness, but at the same time quite strikingly, by placing between the poles of a powerful compound horse-shoe magnet or electro-magnet some part of a flexible conductor, through which a current is then made to pass. Gold strip such as is used for binding hair is well suited to the purpose, and will stand a current of 20 ampères or more if the circuit is only closed for a short time. When the direction of the current is reversed, the strip rushes suddenly across the field to one side or the other.

188. **The left-hand rule.**—All cases of motion of a movable portion of a current-conductor placed in a fixed magnetic field are in accordance with one simple rule, however complicated the relations involved may seem to be in individual instances. By extending the thumb and first two fingers of one hand, so that they are mutually at right angles, we have a means of representing the twofold symmetry which was shown in the last paragraph to characterise the phenomenon. If the hand is turned through half a revolution about the middle finger as axis, the direction in which the thumb points becomes exactly reversed, and the same result also holds when the half-revolution is made about the forefinger as axis.

If we perform all the permutations of the experiment described in the last paragraph, combining each direction of the current with each direction of the field of the magnet, we shall find that there is an invariable relation connecting the three directions: of the current, of the lines of force of the magnet, and of the resulting displacement of the conductor. We have already used the forefinger to indicate the direction of the magnetic lines of force, from north pole to south pole, while the middle finger has been used to indicate the direction of the current. We shall accordingly find that the direction in which the current is displaced, across the direction of the magnetic lines of force, is always that indicated by the extended thumb, provided we are using the fingers of the *left* hand.

This is FLEMING's three-finger rule, which may be formally stated as follows:

If we hold the forefinger of the left hand in the direction of the lines of force of a fixed magnetic field, and the middle finger in the direction of the current, the current-conductor will be urged in the direction of the thumb of the same hand, across the direction of the lines of force. ('Left-hand rule.')

It will be well to verify this rule for all the eight combinations named above. The thumb and two fingers of the left hand, pointing in mutually perpendicular directions, constitute a left-handed system of co-ordinates. In the sequel we shall find it convenient to cover the two fingers and the thumb with cylinders of white, red, and blue paper respectively, corresponding to the directions of the lines of force, the current, and the displacement of the current-conductor. When we come to deal with the phenomena of induction, we shall have to formulate a 'right-hand rule,' which is geometrically the opposite of that now considered.

Our present rule also includes the cases to which the thumb-rule and swimming rule of § 160 are applicable; those cases, namely, where the current is fixed and the origin of the magnetic lines of force movable. Since for every action there is an equal and opposite reaction, it follows that when the current is fixed the bundle of lines of force proceeding from the north pole of the magnet will be urged in the direction opposite to that of the thumb; but the direction thus specified coincides with that in which the thumb of the right hand points when the palm of the hand is turned towards the source of the lines of force—that is, towards the north pole of the magnet. The reader may easily satisfy himself of the correctness of this statement by laying the right hand upon the middle finger of the left in the sense of the current, following the rule given in § 160.

189. Mechanical interpretation of the dynamical forces.—In Chapter II. we explained the mutual action of two magnets, not by supposing that their poles exerted forces of attraction or repulsion on one another, but by means of tensions and pressures in the intervening medium, that is in the field common to the two magnets. We shall now show that the forces which currents and magnets exert upon one another may be deduced from the structure of the resultant field, without any necessity for further assumptions.

If a rectilinear current-conductor is brought into a magnetic field, which for the sake of simplicity we may suppose to be uniform, the lines of force belonging to the two fields will have the same direction on one side of the conductor, and contrary directions on the opposite side, provided the current does not coincide in direction with lines of force of the magnetic field. In accordance, then, with § 123, the effects of the lines of force become strengthened on one side of the current and weakened on the opposite side, and hence the conductor must experience a force tending to move it through the magnetic field. That the direction of the force is that which corresponds to the left-hand rule will be evident when we remember in which sense the current is embraced by its lines of force.

The model, fig. 80, exemplifies very clearly the superposition of two fields in the present case. The two parallel limbs of a wooden framework are marked respectively N (red) and S (blue), and represent for example the poles of a horse-shoe magnet. To represent the lines of force *hh* of the uniform field between N and S, a number of indiarubber tubes are stretched across, their ends being secured to pegs, and arrows being marked upon them as in fig. 39. Within the field thus represented, we lay a model of the system of lines of force surrounding a current, fig. 65. T is the cross-section of the conductor, the direction of the current being towards the observer, and *rr* are the concentric ring-shaped tubes of force. To represent the possibility of sending a current through T, this conductor must be capable of moving parallel to itself along a pair of rails, parallel to N and S (that is, vertical), one in front of the magnetic system and the other behind. The rings of lines of force surrounding the current run counter-clockwise as seen in the figure, corresponding to the direction in which the current flows. Hence the rotational motions are towards the observer through the apertures of the rings, as indicated by the little arrows.

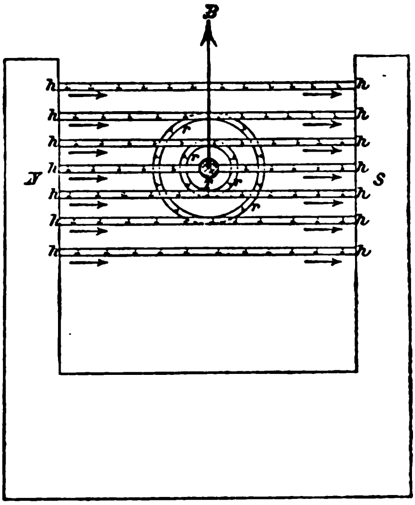

FIG. 80

Above the conductor T, fig. 80, each element of volume is traversed in opposite directions by lines of force due to the two fields, so that the two effects are antagonistic, the resultant energy per unit volume and the pressure across the lines of force being correspondingly diminished. Thus below the conductor there is a pressure acting, which tends to thrust the conductor away, and is not compensated by an equal pressure above. Hence the resultant force on T is directed upwards as indicated by the arrow B. The effect is further increased by those parts of the lines of force which lie more to the two sides, since in the lower parts of the lines the components of the resultant rotational motion are greater than in the corresponding upper parts.

If the forefinger of the left hand be laid upon the lines of force *hh* in the direction (N–S) of the arrows, and if the middle finger be extended in the direction of the current (*i.e.* from the plane of the figure towards the observer), the thumb will point upwards, in the direction of motion B (left-hand rule.)

The special form of the mechanism in motion is here of less importance than the energy content of the various portions of the field, as exemplified by means of the mechanism. For we saw in § 170 that there must be a certain quantity of this field-energy to produce such mechanical forces as those now considered, the resulting displacements causing a change in the number of lines of force embraced by the current. Compare also § 194 below.

190. **Electro-magnetic rotations.**—If we limit the freedom of motion of the current-conductor by constraining one point of it to occupy a fixed position in space, and if we arrange the movable portion of the conductor and the magnetic field so as to be symmetrical about this point, the mutual action between the current and the magnet may be made to produce a continuous rotation.

Of the large number of mechanisms which have been constructed to show this rotation of a movable current-conductor in a magnetic field, we shall describe only a few typical examples, some of which are chiefly of historical interest, while others have important practical applications.

(*a*) *Faraday's rotating wire.*—This oldest and simplest of all such devices is still found the most convenient with currents of moderate strength. A movable piece of conducting wire has an eye at one end, by which it hangs in the field of a fixed magnet, whose axis passes through the point of suspension. The lower end of the wire dips into mercury, and when the circuit of the current is closed the wire rotates round the magnet.

An ordinary cylindrical lamp-chimney G, fig. 81, 24 cm. in length and 4·5 cm. internal diameter, is closed by corks S and T above and below. Through the centre of the lower cork the bar magnet M is stuck so that one of its ends *n* projects to a distance of about 3 cm. into the interior of the cylinder. A conducting wire also passes up through the lower cork into the mercury. Another copper wire passes through T, and ends in an amalgamated loop H. From this hangs the straight piece of wire D, to which a small paper vane is attached. The lower part of the cylinder contains just enough mercury to ensure that the lower amalgamated end of the wire D shall dip well into it. The bar magnet M is fixed in a vertical position in a stand.

If a current is sent through the movable wire D (fig. 81), from below upwards, and if the north pole of the magnet M is uppermost, the movement impressed upon the wire D, when it occupies the position indicated in the figure, will be away from the observer, and the continued action of the two fields upon one another will produce a rotation in the sense of the dotted arrow,—that is, counter-clockwise as seen from above.

(*b*) *Rotating hoop of wire.*—A more symmetrical arrangement may be made by dividing the current, and making it pass down through the two legs of a wire hoop, one on each side of a bar magnet, fig. 82.

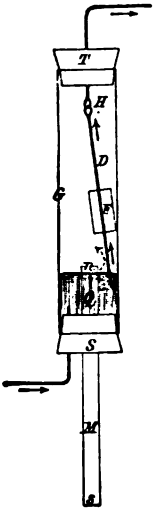

Fig. 81

The pole *n* of a powerful bar magnet or electro-magnet is surmounted by a wooden cap H, round the lower part of which runs an annular groove RR containing mercury, to which the conducting wire Z is connected. The pointed pivot *s*, directed vertically upwards, is fixed exactly in the centre of the cap, and upon it turns the wire hoop BB fitted with the bearing N. The two limbs of this hoop pass vertically downwards, and dip into the mercury channel RR on either side. The upper side of the bearing N is hollowed out and contains mercury, into which dips the amalgamated point of the wire D, held in an ordinary clip. Thus a current can enter the hoop through D.

The current entering at N (fig. 82) becomes divided and flows along the two limbs of the hoop BB down to the mercury in the channel RR, from which it is led away by the wire Z. If it is the north pole *n* of the magnet which projects into the field of the current, the right-hand limb will be urged towards, and the left-hand limb away from the observer, so that the hoop of conducting wire will rotate in the sense indicated by the dotted arrows, that is in the clockwise direction as seen from above. This rotational direction, combined with the (upward) direction of the magnetic lines of force, constitutes a left-handed screw.

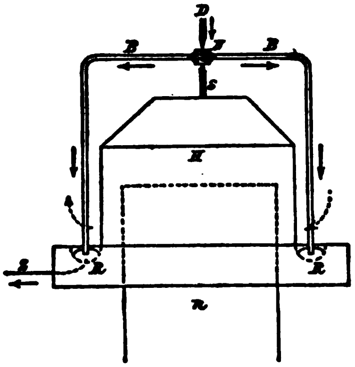

Fig. 82

191. Magneto-electric motor.—The rotational movements experienced by current-conductors in the fields of fixed magnets are applied in the construction of machines which overcome frictional resistances at the expense of the energy of the electro-magnetic field, maintaining their moving parts in continuous rotation and having a capacity for performing mechanical work. This is the principle of magneto-electric motors, or 'electro-motors.' In all technical applications the movements directly produced are rotatory, though in some cases they are converted by suitable mechanism into a progressive or a reciprocating motion.

The steam engine primarily produces only a reciprocating backward and forward motion, which is transformed into a rotatory motion by an intermediate mechanism of connecting rods and cranks. Herein lies the principal advantage of the electro-motor over the steam engine, the former producing directly the desired rotatory motion.

Fig. 83 represents an arrangement which may serve as a model of a magneto-electric motor. Between the two equal and opposite poles N and S, belonging either to different magnets or to a horse-shoe or electro-magnet with suitable pole-pieces, the wire circuit BB is mounted upon the fixed stem T, about which it can rotate. The middle of the upper horizontal portion of the wire has been beaten out until it is rather broad, and indented with a centre-punch, the indentation resting upon the pointed end of the axle T. The lower extremities EE of the wire are crossed over one another, without touching, within the insulating ivory block D. This block can turn freely about the stem T which is fixed in a wooden base H. Sunk in this base are two semicircular channels Q_1, Q_2, into which the wires z_1 z_2 protrude. If mercury is poured into the two channels, there results a horizontal conducting ring whose continuity is broken at two opposite points of its circumference. (Compare the smaller diagram lying below the principal one in fig. 83, and representing the base H as seen from above.) The apparatus is to be placed between the poles of the magnet so that the axis of symmetry of the ring is perpendicular to the lines of force.

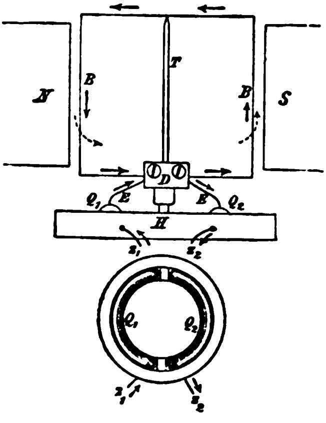

Fig. 83

If there is sufficient mercury in the semicircular channels, it will project so far above the horizontal face of H that the two amalgamated ends EE of the wire circuit dip into it as they pass during the rotation, while the portions of mercury in the two compartments do not flow together across the little wooden partitions.

When the apparatus is in the position shown in fig. 83, a current enters by z_1, passes along the semicircular mercury channel Q_1, to the immersed end of the wire circuit B, up the right-hand side of the circuit, along the top, down the left-hand side, through Q_2 and out by the wire z_2. The action between this current and the system of lines of force passing from N to S causes the circuit to be urged in the sense indicated by the dotted arrows, that is counter-clockwise as seen from above.

The current-conductor continues to experience a rotational impulse under the influence of the fixed magnetic field until it has reached the position in which it embraces the maximum number of lines of force—that is, until its plane has become perpendicular to the direction of the field NS. Since the ring, moreover, possesses some inertia, the kinetic energy which it has acquired will cause it to turn some way beyond this middle position. But in order that the rotation may still be helped forward in the same direction, the field of the current must have its direction reversed. This result is automatically effected; for that end E of the wire circuit which previously dipped into Q_2 passes across the intervening gap so as to reach Q_1, and conversely. thus the current is made to flow up that side of the circuit which it previously flowed down, and the field of the magnet seeks to turn the circuit right about, which corresponds to a continuation of the existing motion in the same sense. At the next time of passing through the mean position (dead point) the direction of the current is once more automatically reversed in the movable part of the circuit; and so on.

If, instead of a single circuit of wire, we arrange a large number of such circuits upon one axis with their planes intersecting one another, the result is a drum-shaped surface entirely built up of these circuits, with currents flowing up one side and down the other. Such a drum when provided with a suitable commutator and placed in a magnetic field, is set in rotation, the separate windings, which follow closely one after another, all helping to increase the effect ('drum armature').

In order to gather as many lines of force as possible through the windings, the wires are wound upon an iron drum, not made in one continuous mass but built up of thin plates of iron, insulated from one another and closely bound up together ('laminated armature'). The reason for this 'lamination' will appear later. We need not speak here of the special arrangements which serve to reverse the current in the separate windings just at the right instant. They will be described in connection with dynamo machines, which are exactly the opposite of electro-motors in action.

192. **Rotation of radial currents.**—In all cases where we have spoken of the axial system of lines of force of a galvanic current, we have assumed that the annular lines of force of these systems are, as it were, bound up with the material of the current-conductor, just as we saw, in the case of ordinary magnets, that the issuing and re-entering lines of force are definitely attached to the steel. The correctness of our assumption is shown in a specially instructive manner when we consider those continuous rotations which occur in the case of so-called radial currents, flowing within the fields of fixed magnets.

Experiment 65.—A circular disc of copper has its edge amalgamated, and is mounted upon a horizontal axle about which it can rotate freely, the lowest point of its circumference dipping into a drop of mercury in a hollow placed beneath. A strong current enters the disc at its centre and passes to the drop of mercury below, from which it is led away by a conducting wire. This current flows principally along that radius of the disc which terminates in the mercury, producing around it an axial system of lines of force. If the lines of force of a powerful horse-shoe magnet are made to pass perpendicularly through the lower part of the disc, this latter will be set in rotation (Barlow's wheel). Those parts of the wheel through which the current flows will be driven out of the magnetic field in accordance with the left-hand rule. As soon as this happens, however, they cease to be traversed by the current, a new portion of the rim of the disc becoming immersed in the mercury; but the deflecting influence is now exerted on that radius of the wheel which has newly assumed the function of current-conductor, and which in consequence is also driven out of the field, and so on, the result being a continuous rotation of the disc.

Experiment 66.—The same effect is still more strikingly shown when a flat circular dish filled with mercury is placed upon one pole of a powerful electro-magnet, the internal margin of the dish being covered with an annular strip of sheet copper. If a metallic current-conductor be dipped into the middle of the mercury, the current being led away from the annular copper strip by a second conducting wire, the current spreads out radially from the centre, and is uniformly distributed in all horizontal directions through the mercury. Since the lines of force of the magnet pass upwards or downwards through the entire movable mass, the whole will acquire a rotatory motion, whose direction is given by the left-hand rule.

In the centre the rotation is strongest, for there the annular lines of force due to the current are most closely packed together. The level of the more central portion of the mercury surface becomes depressed, owing to the action of 'centrifugal' forces. The phenomenon may be rendered visible to an audience by allowing flat corks, carrying little candles, to float on the surface of the mercury.

In this experiment, as in those previously described, a commutator should be introduced into the circuit of the current, so that the direction of the current may be reversible, as well as that of the magnetic field. We have thus the opportunity of applying the left-hand rule in cases of the most varied kind.

193. **Movements of a flexible conductor in the field of a bar magnet.**—The electro-magnetic actions assume a peculiar form when the current-conductor is flexible and is allowed to hang freely near to a fixed bar magnet, and parallel to

it. An arrangement suitable for this purpose is due to Helmholtz.

To a square base-board B (fig. 84) is fastened a wooden stem H, supporting by means of a brass clamp the strong bar magnet M, with poles *n* and *s*. Also attached to H is a thick copper wire D, which arches upwards as shown in the figure, and ends, vertically above the magnet, in a piece bent downwards. The extremity of this piece, which should not be too near to the pole *n*, is split so as to receive the end of a thin gold strip SS, which is firmly gripped by it, and hangs close beside the magnet reaching fully down to the base-board B. A wire conveying a current is attached to the lower end of the gold strip by means of a binding screw, a corresponding wire being attached to D, and a commutator being placed in the circuit.

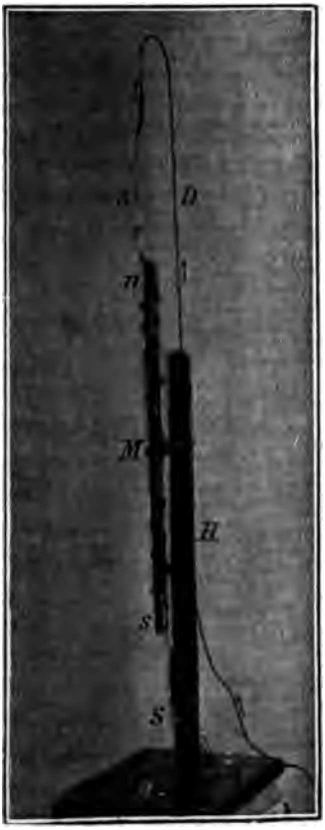

Fig. 84

Experiment 67.—Let the gold strip SS (fig. 84) be supported at the middle so that it hangs loosely in the neighbourhood of the two ends of M, and then let the circuit of the current (about ten ampères) be closed. The strip becomes wound round the upper and lower portions of the magnet so as to form a continuous helix. If the direction of the current be reversed, the strip becomes unwound, and rewound in the opposite direction. The unwinding and rewinding arising from repeated reversals of the current are especially striking. Here again, if the polarity of the magnet has been determined, the direction of the force acting upon each element of the movable conductor may be deduced from the left-hand rule.

For example, we find on applying the rule, that at the upper end of the rod, where the lines of force leaving the pole *n* are proceeding towards the observer, the strip conveying a current in the direction of the arrows, when in front of the magnet M will be urged towards the left (direction of the thumb), when behind the magnet to the right. This is apparent from the figure. At the lower end, that portion of the strip which lies in front of the magnet will be urged towards the right, that which lies behind towards the left.

194. **Quantitative law of the mechanical action upon a movable current-conductor in a stationary magnetic field.**—We have now followed out qualitatively in a variety of cases the action of a magnetic field upon a movable conductor. We saw that the left-hand rule could always be applied to determine the direction of the resulting motion, and we even deduced the law itself from a consideration of the symmetry which characterises the interaction of the fields concerned. We must now formulate the quantitative relations which hold good in such cases.

In all cases where quantitative relations between measurable quantities are concerned, the most general expression of the relation is that derived from the law of energy. From the expression idN, for the electro-magnetic energy of the field of a current (§ 170), by making use of the principle of energy, we may calculate the mechanical force P exerted upon a current-conductor by a magnet, owing to the mutual action of their respective fields.

In fig. 80, for example, let us suppose the rails, on which slides the movable conductor T, to run vertically upwards, and to have their upper ends connected to conducting wires so that they form part of a closed circuit. Thus when T moves in the direction of the arrow B, magnetic lines of force of the field NS which were previously embraced by the circuit will cease to be so embraced, so that the number of lines N threading through the circuit becomes continually less. Thus if the current strength i is maintained constant, the electro-magnetic energy is diminished by the amount idN when dN lines of force of the fixed system NS leave the embrace of the circuit, that is, when dN such lines are cut through by the sliding conductor T. We may suppose that this happens when T has moved forward through the distance ds. In place of the energy idN some other form of energy of equal amount must be produced, in accordance with the principle of energy. If P is the value in dynes of the force acting upon the conductor T, this force does work Pds during the displacement ds. Hence if no loss of energy takes place, the work thus gained must be equal to the loss of electro-magnetic energy; that is

$$Pds = idN \quad \ldots\ldots\ldots \quad (34\,a)$$

or

$$P = i\,\frac{dN}{ds} \quad \ldots\ldots\ldots \quad (34\,b)$$

In the case represented in fig. 80, the field is uniform, the sliding conductor is straight and perpendicular to the lines of force, and the displacement of the conductor is perpendicular to both these directions. The expression for dN can then be very easily calculated. If $\mathfrak{H}$ is the field intensity, l the length of the conductor T (measured between the rails on which this conductor slides), the area swept out by T when it moves through ds cm. perpendicularly to its length is lds cm.^{2}, and through this area pass $\mathfrak{H}lds$ lines of force of the field. These $\mathfrak{H}lds$ lines of force are cut through by the movement of T, so that they cease to be embraced by the circuit. We have thus $dN = \mathfrak{H}lds$, and

$$\mathfrak{H}P = i\mathfrak{L}l \quad \ldots\ldots\ldots \quad (35)$$

If a rectilinear conductor of length l *forms part of a circuit conveying a current of* i *deca-ampères, and is placed*

perpendicularly to the lines of force in a uniform magnetic field of intensity $\mathfrak{H}$, the force acting upon the conductor is equal to the product of the three quantities named.

This law is of the greatest importance in the construction of electro-motors, since it serves to determine the capacity of the motor for doing work.

If the three directions in question are not mutually perpendicular we shall be concerned with projections, so that circular functions will appear in the formula. If the direction of motion ds makes an angle θ with the direction of the lines of force, the change in the number of lines of force embraced by the circuit, arising from the displacement ds of the conductor, is

$$dN = \mathfrak{H} l . ds \sin \theta.$$

If further the rectilinear conductor is inclined at an angle ϵ to the lines of force, l must be replaced by the product $l \sin \epsilon$, so that we have as the general expression for the work done upon a conductor by the stresses in the electro-magnetic field

$$i \mathfrak{H} l . ds \sin \theta \sin \epsilon,$$

the force being accordingly

$$P = i \mathfrak{H} l \sin \theta \sin \epsilon \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35a)$$

[It must be remembered that work has to be continually expended to maintain the current i in the circuit, so that the problem does not stand upon the same footing as if no energy were supplied from sources external to the system considered. As a matter of fact, a circuit in which a current is *maintained constant* by external means always tends to assume that position and configuration in which the electro-magnetic energy is a *maximum*.]

CHAPTER XII

BIAXAL AND MULTIAXAL MAGNETIC FIELDS OF FORCE

So far we have considered only the field of a single galvanic current, and its combination with the fields of one or more magnets. This combination of fields forms the basis of the important group of phenomena classed together under the name of electro-magnetism. We pass now to the consideration of those fields which are due to the simultaneous presence of two or more currents. Each such current constitutes the axis of its own system of lines of force. The structure of the field is accordingly determined by the simultaneous existence of the several axes, so that in this sense we may speak of 'biaxal' or 'multiaxal' magnetic fields.

The coaxal system of lines of force due to a current is equivalent to that arising from a suitable magnetic distribution. Hence it follows that currents must exert an action on magnets, and magnets on current-conductors. This action, again, must take place in just the same manner when the magnets in question are replaced by current-conductors, for example by solenoids or bobbins. That is, since magnets act upon magnets, there must also be an action of currents on currents. This result was established by Ampère in 1820, that is, immediately after the announcement of Oersted's discovery of the mutual action between currents and magnets. According to the doctrines of that time a current was due to the flowing of a certain something, which could be collected in a stationary condition by rubbing bodies together, and which was called electricity. The actions exerted upon one another by rubbed bodies in virtue of their 'electric charges' were called by Ampère 'electrostatic,' the actions arising from 'current electricity' being called 'electrodynamic,' just as that branch of mechanics which deals with the phenomena of motion was called dynamics, and that which deals with the conditions of rest, statics. Thus the whole subject has received the name of 'electrodynamics.'

When we take the conception of lines of force as the basis of our treatment, the present chapter follows quite naturally from the last; the system of magnetic lines of force upon which we there supposed an axial system to be superposed has simply to be replaced by a system coaxal with another conductor.

A.—Electrodynamic action between two different current-conductors

195. Field of two same-way-directed parallel currents.—Taking the line-of-force diagram as our starting point, we can see immediately what phenomena must occur under any given conditions. Suppose that we have given two rectilinear current-conductors parallel to one another and not at too great a distance apart. Each of these is to be an axis of force—that is, it is to be joined to a pair of terminals in a determinate sense; and in the first place we shall suppose this sense to be the same for the two conductors.

The same current may be used for both conductors if their contrary ends are joined by a metallic wire, so widely bowed out as to be without influence on the portion of the field under investigation, the portion, that is, which lies between the parallel conductors.

We suppose the conductors arranged in a vertical position, the plane of the diagram being taken at right angles to their length, that is in a horizontal plane. In each conductor the current-strength is the same.

In this experiment we shall make use once more of the apparatus described in § 145, and represented in fig. 47. Near to the first wire L_1, and parallel to it, a second similar wire L_2 is arranged, its ends being immersed in the second pair of mercury cups, while corresponding holes in the board C and the paper P allow it to pass through. The wire H_1, which leads from the positive terminal, is dipped into the left-hand upper cup, so that the current passes downward through the left-hand conductor L_1 (compare the smaller figure to the side). Into the left-hand cup in the lower block E_2 dips the end R of the auxiliary wire, its upper end dipping into the right-hand cup of the upper block E_1, whence the current passes downward again through the conductor L_2, and into the right-hand cup of E_2 dips the wire H_2, which leads the current away to the negative terminal K_2. If a commutator is introduced into the circuit, we may at pleasure reverse the direction of the current, so that it flows upwards through the two parallel conductors.

If the wires are afterwards made to pass through different holes in the board C, we may thus obtain the line-of-force diagrams corresponding to different distances between the wires, the current-strength remaining the same.

If the two current-conductors are far apart (fig. 85) they hardly produce any appreciable disturbance of one another's fields (fig. 85); round each of their cross-sections SS run circular lines of force, as if the second current were not present. But if the two conductors are brought nearer together, so that each of the two axial fields extends into the range of the other, the course of the lines of force becomes modified (fig. 86).

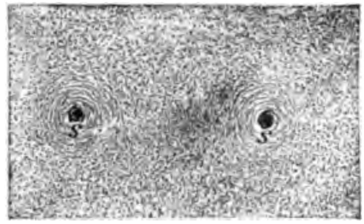

FIG. 85

Each of the two current-conductors S_1, S_2, considered by itself, has a coaxal system of lines of force, the form of which in the immediate neighbourhood of the conductor is maintained without sensible disturbance, except for a small blank space in the innermost part of the field where the tension tending to shorten the lines of force has been so great as to drag the iron particles away into contact with the surface of the conductor. If the currents are both flowing downwards, the lines of force will run clockwise around S_1 and S_2. Between the two conductors, their annular lines of force are in contrary directions, but as we pass to more distant points, the agreement of direction between lines of the two systems becomes closer and closer. In accordance with §§ 88 and 123, the influences exerted by S_1 and S_2 will more or less completely neutralise one another at points between S_1 and S_2, while in more outlying parts of the field they will strengthen one another.

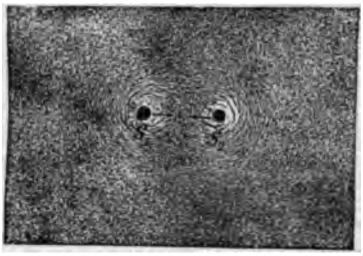

FIG. 86

There will be one curve in the diagram formed like a lemniscate or figure of eight, and embracing both conductors. The lines running just outside this curve are somewhat sharply bent inwards above and below. Exactly in the middle, between S_1 and S_2, there is a double point where the two halves of the figure-of-eight curve meet. Within these two halves again, the lines of force are in the form of ovals, with considerably elongated ends turned towards one another. As we approach the conductors more closely, the form of the lines becomes more and more nearly circular. The lines at a greater distance from the conductors become less and less indented above and finally assume an elongated oval or elliptical form. At greater distances still, the two fields become so blended together that they might be taken for the field of a single current of twice the strength of either.

196. **Electrodynamic action between same-way-directed parallel currents (attraction).**—The tension along the magnetic lines of force (fig. 86) must tend to draw the two conductors closer together. Each annular line of force tends to shrink together like a stretched rubber band; thus it presses upon the inner annular lines and these finally upon

the innermost oval and circular lines. These last, however, are close against the current-conductors themselves, and transmit to them the forces arising from stresses in the medium. Thus the two conductors experience forces urging them together.

Like parallel currents attract one another. This mode of expression, based upon the older view of the phenomenon, is retained for the sake of shortness, although we look for the seat of the action in the medium surrounding the wires, referring it to stresses in the field—tensions along and pressures across the lines of force. The mechanical forces exerted upon the conductors are represented in fig. 86 by two arrows pointing towards one another.

The resultant field of the two currents is the seat of electromagnetic energy, the amount of which per unit of volume varies from point to point, since the field-intensity is not everywhere the same. If the differences of intensity are compensated by the action of other (mechanical) forces which constrain the conductors to remain at rest, no work is done. If, however, the compensating action is withdrawn, an 'electrodynamic' phenomenon results, the wires move towards one another, part of the electro-magnetic energy of the field being transformed into palpable kinetic energy.

[If the currents are maintained constant, the energy of the field will be *increased* by the nearer approximation of the conductors, the supply of energy being derived from the battery or other agent which maintains the current.]

197. **Field of two oppositely directed parallel currents and their electrodynamic action on one another (repulsion).**—The two neighbouring parallel rectilinear conductors must now be made to convey currents in opposite directions.

The line-of-force diagram may again be best obtained by means of the apparatus fig. 47. The ends RR of the auxiliary wire are removed from the mercury cups, while H_1 (for example) is made to dip into the left-hand upper cup (as seen from in front). Thus the current from K_1 passes downwards along the wire L_1. The wire hoop *b* is arranged as a bridge connecting the mercury cups of the lower block E_2, so that the current rises in the conductor L_2, and is finally conducted away to K_2 by the wire H_2, which is turned upwards and made to dip into the right-hand upper cup. On commutation of the current the directions in the two neighbouring conductors still remain opposite to one another.

The line-of-force diagram for this case is represented in fig. 87. S_1 and S_2 are the cross-sections of the two conductors. If the current along S_1 is from above downwards through the plane of the diagram, its lines of force are in the clockwise direction. In this case, the current along S_2 flows from below upwards through the plane of the diagram, and its lines of force, as seen by the observer, run counter-clockwise. Hence in the middle between S_1 and S_2, the lines of force which lie side by side are in the same direction. The tensions and pressures which they would separately exert are added together. The annular tubes of force are densely crowded together, the pressure which they exert upon one another being so considerable that the rings belonging to the two currents are widely displaced from their concentric arrangement, and driven further outwards. Between S_1 and S_2 the distances between consecutive tubes are less, while outside the line S_1 S_2 they are greater than in the case of a single undisturbed field (compare fig. 48). This pressure across the lines of force tends to drive the two current-conductors asunder, as indicated by the arrows pointing right and left in the diagram. If the conductors are free to move, the distance between them will be increased by the action of the stresses in the field: '*the conductors repel one another*.' This electrodynamic 'repulsion' is the exact opposite of the phenomenon described in § 196.

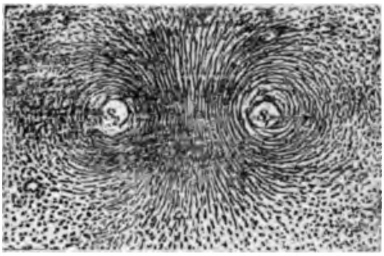
FIG. 87

If the sectional diagrams of the fields of force described in §§ 195, 197 are formed afresh after the direction of the current has been reversed, we may thus satisfy ourselves that the direction of the parallel currents in space has no influence on the result, but that the agreement or opposition of direction of the two currents determines the form of the resultant biaxal field.

198. **Fundamental electrodynamic experiments.**—The mechanical forces arising from the distribution of lines of force may be very easily demonstrated by means of various arrangements, of which we proceed to describe some typical examples.

(*a*) *Ampère's table.*—Ampère supported wire rectangles upon points dipping into little cups filled with mercury. If these cups are connected to a source of current, and if a second conductor (flexible, for example) conveying a current of about the same strength is brought near, attraction takes place in the case of same-way-directed currents, while on reversing the direction of either one of the currents, the effect becomes one of repulsion.

To remove the slight disturbing effect which arises from the action of terrestrial magnetism, it is convenient to use a double rectangle, that is a wire bent into the form of two rectangles lying side by side and traversed by the current in opposite cyclic directions. The bundles of lines of force which thread through the two rectangles are of equal strength and exactly opposed to one another in direction, so that the action of the earth's magnetism upon the one is just compensated by its action on the other. the double rectangle being astatic. The electro-dynamic experiment may then be performed upon one vertical or horizontal side of the rectangle, the direction of the current along that side being indicated by the attachment of an arrow.

(*b*) *Suspended coils.*—It is very convenient to use two coils of thick insulated aluminium wire whose wide windings are easily followed by the eye, and which are suspended close to one another by pairs of gold threads. In the circuit of each a commutator is interposed and a strong current (4 to 6 ampères) is made to circulate, its direction being best indicated by means of arrows attached to the circuit. If the currents are in the same direction, the two coils strongly attract one another and remain in close contact, while if one of the currents is reversed in direction, repulsion takes place.

Influence of the permeability of the intervening medium.—Since the mechanical forces arise from the stresses which accompany the lines of force, their magnitude must necessarily be increased when a greater number of lines is collected together in the interior of the coils.

Experiment 68.—While repulsion is taking place between two coils, in which oppositely directed currents of equal strength are circulating, let a thick round bar of soft iron be thrust through the apertures of the coils without touching them ; the divergence of the suspending threads will be found to increase. If when the currents agree in direction the points of suspension are so far apart that the mutual attraction of the coils cannot produce actual contact between them, the introduction of the iron core will greatly increase the deviation from the position of rest, so that a much greater component of gravity is needed to balance the attraction.

The mutual attractions and repulsions of the coils may be regarded as arising from the bundles of lines of force with which they are associated when currents are made to circulate in them, and in this connection we may refer once more to the model in fig. 53. From the disposition of the lines of force it will be seen that when currents are sent in the same sense round coils whose windings are parallel, opposite polarities are turned towards one another, the lines of force which emerge from the one coil passing directly into the other (attraction) ; oppositely directed currents, on the other hand, cause the polarities which face one another to be alike (repulsion).

199. **Mutual electrodynamic action between the separate turns of the same helix.**—When we examined the course of the lines of force through an open solenoid (fig. 73) we saw that even the separate turns of the same current-conductor are encircled by lines of force as are the parallel wires S_1 S_2 of fig. 86, being likewise traversed by same-way-directed currents. Although the innermost (annular) lines

of force embrace only the separate turns of the conductor, the outer lines coalesce with those belonging to neighbouring turns. As, in the case of different currents flowing in like directions, these lines of force tend to draw the two conductors together, it follows that there will be a similar electrodynamic action between the neighbouring turns of the solenoid which are parallel to one another, and traversed by currents in the same sense. The whole helix has a tendency to become shorter, the separate turns of wire appearing to attract one another. This effect is shown very strikingly in ROGET's spiral.

Experiment 69.—A long helix is made of thin copper wire, and consists of a single layer of turns wound as closely together as possible. The helix is hung up by one end, the lower end being loaded with a small weight. The end of the copper wire is amalgamated and dips vertically downward into a mercury cup, which is joined to one of a pair of terminals, the other terminal being joined to the upper end of the helix. As soon as the circuit of the current is closed, the turns of the helix become drawn together. Since the uppermost turn is fixed, the lower turns are thus lifted up, so that the amalgamated end of the wire is raised out of the mercury; the lines of force and the pull arising from them cease to exist, the helix falls back under the action of gravity, and the end of the wire becomes once more immersed. Once more the phenomenon of the current is repeated, and the contractile stress acts along the direction of the field, the same cycle of changes occurring again and again. The end of the spring is thus alternately immersed in the mercury and withdrawn from it, its movements being accompanied by the rhythmical crackling of the vivid sparks which occur on breaking the circuit.

200. Action of different parts of the same current-conductor upon one another.—The older theories sought to analyse all observed actions between finite systems into actions between infinitesimal elements of those systems. Thus the forces exerted by current-conductors upon one another were resolved into those arising from so-called current-elements, to which was ascribed a mutual action according to a definite law, the action between finitely extended conductors being calculated from these elementary actions by addition (integration). If however we confine our attention to the mutual actions of finite systems, such as alone occur in nature, we may apply our doctrines to the elucidation of certain experiments which were designed to show the existence of such actions of current-elements on the remaining portions of a circuit.

Experiment 70 (*De la Rives' floating hoop*).—Two parallel grooves are cut in a board, at a small distance apart, and are filled with mercury. The grooves must be of such a breadth that in the middle of the reflecting mercury surface there is no appreciable capillary curvature. Current-conductors are connected to these two grooves, which are also placed in conductive connection by means of a hoop of copper wire, this latter having two long horizontal parallel limbs which rest upon the mercury surfaces. The entire hoop is covered with an insulating layer of sealing-wax, except at the extreme ends of the two limbs which are bent downwards, and expose an uncoated copper surface to the mercury. Thus it is through these extremities that the current flows.

When the current circulates, the hoop swims away, in whichever sense the current may be. Thus it would seem as if the portion of the current in the hoop were repelled by the portion in the fixed parts of the circuit.

According to our theory, however, the phenomenon is to be explained by the pressure across the lines of force which are embraced by the path of the current. Every such circuit embraces a number of lines of force determined by its shape and size, by the strength of the current, and by the nature of the surrounding medium, § 155. These lines of force press outwards against the path of the current (compare experiment 54, § 150). If anywhere along this path there is a place where the conductor can easily give way (like the hoop in the present case) the movable portion will be driven outwards by the pressure of the lines of force.

Thus it might appear as if this part of the conductor were repelled by the remainder.

201. **Behaviour of crossed currents.**—If the two conductors are not parallel, but make any angle with one another, their mutual action may be most simply inferred from a consideration of the lines of force in each of four dihedral regions, into which we may suppose the field divided. The rectilinear conductors in general cross one another obliquely, the shortest line which can be drawn from one to the other being perpendicular to both. Let two planes be drawn, each passing through this common perpendicular and one of the conductors. Thus the field is divided into four dihedral compartments, in each of which the direction of the lines of force is determined by the directions of the currents. Let us fix our attention upon volume elements in outlying parts of the dihedral regions, where it is easier to follow the composition of the lines of force belonging to the two systems. In the two compartments where the currents are flowing both towards or both away from the point of crossing, the disposition of the lines of force will be very similar to that in the biaxal field of parallel *same-way directed* currents; hence these portions of the conductors will be urged together. On the other hand, the disposition of the lines of force in the other two dihedral regions will be similar to that in the field of parallel *oppositely-directed* currents, so that the corresponding portions of the conductors will be urged asunder. In other words, the pressure due to the lines of force seeks to widen the dihedral angle of these regions, thus adding to the effect in the two other compartments of the field, where angle tends to become smaller. The total effect is thus a tendency *to bring the crossed conductors into parallelism.*

These conclusions may be verified by experiments, performed, for example, with Ampère's table. The phenomenon, then, is also seen to be a consequence of the disposition of lines of force, no assumption being needed as to attractions or repulsions between elementary portions of the current-conductors.

202. **Currents in perpendicular directions; continuous rotation of one current-conductor under the influence of another.**—Let there be a straight conductor, such as a channel filled with mercury, in connection with which is another straight conductor in a perpendicular direction; for example, a vertical copper wire, whose lower end dips into the mercury, and which is encircled by lines of force due to a current flowing in it. Let us consider the disposition of the lines of force in a plane passing through the two conductors. The annular lines of force due to these two conductors are perpendicular to the plane in question and to one another; in one of the angular compartments into which we may suppose the field divided (as in the last paragraph) the two sets of lines of force intersect the plane in the same direction, and here accordingly they mutually strengthen one another's effect, this being the case in the regions along whose boundaries one current flows towards the point of intersection, and the other current away from it. In the adjoining regions where both currents flow towards the point of intersection, or both away from it, the two sets of lines of force pass through our plane in opposite directions, so that their effects are antagonistic, and the pressure across their direction is correspondingly less. The vertical conductor must therefore experience a force perpendicular to its length, and urging it away from the region first mentioned. If it is free to move parallel to itself, like the wire dipping in the mercury channel, it will be driven along the other conductor, towards that side where the two currents have the same direction relatively to their point of intersection.

This cross-pressure may be applied to produce a continuous rotation of one current-conductor under the electrodynamic influence of another, just as under the influence of the field of a fixed magnet. A copper hoop is capable of rotating upon a fixed pivot, whose axis is vertical, the two limbs of the hoop being bent vertically downwards, and dipping into a circular channel of mercury, from which a current flows through them in all positions, either ascending in both limbs, or descending in both. The circuit is completed through the pivot. Surrounding the vertical limbs of the hoop is another circuit consisting preferably of several turns, and having the plane of its windings horizontal. The mutual action of the two fields produces a continuous rotation of the copper hoop, in the one sense or the other, according to the manner in which the directions of the two currents are related to one another.

The action in the vertical tangent plane which we have considered outweighs any effect upon the more distant, bent portions of the current-path.

203. **Magnitude of the electrodynamic action.**—At each point of a biaxal or multiaxal magnetic field the resultant effect is to be found by compounding together the effects due to the separate conductors. But the field-intensity due to a current is at each point proportional to the current-strength (§ 165). In a magnetic field formed by the superposition of two such fields [the field-intensity is the sum of two terms, each proportional to the strength of one of the currents. The stresses at each point are proportional, however, to the *square* of the corresponding field-intensity, and accordingly contain terms proportional to the *product* of the two current-strengths. It is to these 'product terms' in the resultant stresses that the mutual electrodynamic action between the circuits is due]. The mechanical forces exerted upon the current-conductors in the resultant field depend upon the distribution of the lines of force, and especially upon their number; they are proportional to the product of the two current-strengths, or in the case of two portions of the same circuit acting upon one another, to the square of the current-strength.

W. Weber has established the accuracy of this result by a series of very careful experiments with coils conveying currents, one of the coils being bifilarly suspended (bifilar electrodynamometer).

B. Electrodynamic flux-of-induction diagrams

The diagrams of uniaxal magnetic fields described in § 172 may be combined with other diagrams of the same kind, just as the diagrams of two or more magnetic fields were combined to represent the resultant of those fields. Thus we obtain a representation of multiaxal or electrodynamic fields.

204. **Combination of the diagrams of two axial fields of force.**—Let the diagram for each current be prepared as in § 172, and then let the two diagrams be superposed. We thus obtain the distribution of lines of force in the field of two currents, whose directions may be either the same or opposite. The considerations involved in this case are quite similar to those already mentioned in connection with the production of a bipolar magnetic field from two unipolar fields. They lead to the following rule: In a plane at right angles to the axes of force, construct the line-of-force diagrams for the two currents (the permeability of the surrounding medium is assumed equal to unity). Thus the diagram is divided up into quadrilateral figures, through each of which a diagonal has to be drawn. In the case of currents whose direction is the same, the lines of force of the two systems encircle their axes in the same sense, and the diagonals are to be drawn as in figs. 89 and 90, Plate II. When the currents are in opposite directions, the lines encircle their respective axes in contrary senses, and the diagonals are to be drawn as in figs. 91 and 92.

In the former case the lines of force in the immediate neighbourhood of the axes A_1 and A_2 are in the form of separate rings, and as we follow the system of lines further out, we see that they pass through an intermediate form resembling a lemniscate to curves which are more and more nearly circular. This case is represented in the iron-filing diagram, fig. 86. In the second case the lines of force surrounding the two conductors are distinct, and never coalesce, but press upon one another so as to be displaced further from the middle of the field, the resulting diagram being as shown in fig. 87.

The correctness of the diagonal rule will be evident if we remember a property characteristic of the diagram for a uniaxal field; namely, that at each point the field-intensity is inversely proportional to the distance between the nearest lines of force (compare § 173). In fig. 88, let r_1', r_1'' be two consecutive lines of force of the system A_1, distant a_1 from one another; r_2', r_2'' two consecutive lines of the system A_2, whose distance apart is a_2. Suppose also that we have to deal with currents in the same sense (both passing for example from above downwards through the plane of the diagram), so that the annular lines of force run in the same cyclic (clockwise) direction. If the lines are drawn at sufficiently small intervals, we may regard the sides of the resulting quadrilateral figures as straight, and these figures themselves as parallelograms. At a point P in the field, let the magnetic forces due to the two separate systems be denoted by $\mathfrak{H}_1$, $\mathfrak{H}_2$, their directions being tangential to the circles r_1', r_2'' at the point P. If the quadrilateral is small, its sides lie very nearly along the lines of action of $\mathfrak{H}_1$ and $\mathfrak{H}_2$.

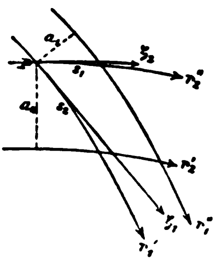

FIG. 88

In accordance with § 173 these magnetic forces are inversely proportional to the distances between the respective pairs of lines of force, so that

$$\mathfrak{H}_1 : \mathfrak{H}_2 = \frac{1}{a_1} : \frac{1}{a_2} = a_2 : a_1.$$

Again, from the similarity of the corresponding triangles the ratio between these distances is equal to the ratio between the sides of the parallelogram, that is $a_2 : a_1 = S_2 : S_1$. Hence th resultant of the two magnetic forces is along the line joining P to the opposite corner of the parallelogram.

Here again the distance between consecutive lines of force in the resultant field is at each point inversely proportional to the resultant field-intensity, as may be seen from considerations quite similar to those which we encountered when combining the fields of two magnets. When a field is to be formed by combining two uniaxal fields, the representation of the lines of force may also be accomplished by means of the following rule:

In each of the two original simple fields let the lines of force be numbered, the innermost circle of radius r_0 having the index $p=0$, the next in order the index $p=1$, and so on ; we have then to combine any line of force of the one system—say that whose index is m—with some line of the other system—for example that line whose index is n. In accordance with § 172 $r_m=e^{m/2i}$, $r_n=e^{n/2i}$. If the two fields are in the same (cyclic) sense, we pass from the point P (fig. 88) where the m^{th} and n^{th} circles of the respective systems intersect, to the point of intersection of the $(m+1)^{th}$ circle of the first system and the $(n-1)^{th}$ circle of the second. That is to say, we join together those two corners of the parallelogram for which the sum of the indices has the same value $m+n$. Thus each of the resultant lines of force, in the case of same-way directed uniaxal fields, corresponds to some determinate value $m+n$ of the sum of the indices. To obtain the lines of force of the resultant field when the original fields are oppositely directed, we must pass from a point of intersection, such as (r_m, r_n) to intersections where both indices are higher or both lower, the difference $m-n$ having a constant value along each of the resultant lines of force. The separate lines of the resultant system are in this case distinguished by different values of the number $m-n$.

205. Properties of biaxal flux-of-force diagrams.—From our construction it follows that in each of the diagrams in figs. 89 to 92, Plate II., the flux of force between each pair of consecutive lines is unity. The canals or tubes of force are supposed to be limited above by a second plane parallel to the plane of the diagram and at a distance of 1 cm. from it. Here again, as in the case where we combined the fields of two magnets, the field-intensity at any place may be immediately deduced from the distance between two consecutive lines at that place. If the deca-ampère is taken as the unit for the measurement of current-strength, and the centimetre as the unit of length, the field-intensity is equal to the reciprocal of the number of centimetres in the distance measured.

(*a*) *Diagram for same-way-directed parallel currents.*—Figs. 89 and 90. The latter of these figures represents a different but similar system of lines of force on a larger scale. The arrows by the conductors A_1 A_2 are intended to indicate that the constituent fields follow the same cyclic direction. The special features which characterise the disposition of the lines of force are much more clearly shown in these diagrams than in the direct iron-filing figure (fig. 86), which is not altogether easy to follow, especially in its more central part. It will be seen how the inner lines of force are elongated to the form of ovals, whose narrower ends are directed towards one another from either side of the field, and approach one another more and more nearly as we pass to the larger ovals. The characteristic features of the diagram are the indentation of the lines which have become fused together, and the way in which these lines continually approximate to the circular form as the distance from the conductors increases.

Along each line, in accordance with the investigation in the last paragraph, $r_m \times r_n = e^{\frac{m+n}{2i}}$ = constant. Lines whose equations are of this form are called lemniscates. It follows from our construction, and from the equation obtained, that for all points on any one of these lines the product of the distances from the two fixed points A_1 and A_2 has the same value.

(*b*) *Diagram for oppositely-directed parallel currents.*—Fig. 91 shows the course of the lines of force through a somewhat wider range than fig. 92, which represents the central portion of the field on a larger scale. As in the component fields, the lines in the resultant field are all circles, their centres, however, being displaced further and further out along the continuations of the line A_1A_2. The central line of force, which is straight, and divides the field symmetrically, may be regarded as belonging to the circumference of a circle of infinite radius. Every other line cuts the line A_1A_2 in two points, one nearer to the conductor, and lying between A_1 and A_2, the other more remote from the conductor and lying in the outer part of the field.

In the middle, between the two wires, the lines of force are much more closely packed together. In the figure, for example, the field-intensity is increased from 1/0·26 (reciprocal of the distance in cm. between two lines of force in the simple field) to 1/0·13 (corresponding quantity for the resultant field); that is, in the ratio 2 : 1.

Along each of these lines we have

$$\frac{r_m}{r_n}=e^{\frac{m-n}{2i}}=\text{const.}$$

The system of lines represented by equations of this form being a family of circles. The straight line A_1 A_1 is divided harmonically by each of the circles.

It will be well also to construct line-of-force diagrams for currents of unequal strength, the process following the same rules as in the case of equal currents. MAXWELL, to whom we owe these and other similar constructions, has also mapped the lines of force for a grating consisting of many parallel wires.

206. Diagram for a circular current with a central magnetic needle.—The diagrams of multiaxal fields, like those of uniaxal fields, may be combined with the diagrams of the fields of magnets, thus furnishing an instructive representation in cases which would be difficult to follow analytically. As an example we shall take an arrangement which is of especial practical importance; a magnetic needle being placed in the field between two conductors, and being free to turn in a plane perpendicular to those conductors. We shall suppose the conductors to be rectilinear and parallel, and traversed by currents in opposite senses. The resulting diagram does not differ essentially from that which would be obtained if the same current were made to pass round a circular conductor, with the magnetic needle at its centre. This latter arrangement is the one adopted in the tangent galvanometer.

Suppose that at A′, fig. 93, a current is flowing upwards through the plane of the diagram, the conductor bending across the paper from left to right, and passing down again through A. Round A′ and A draw the concentric circles representing the lines of force; then draw diagonal lines joining those points of intersection for which the differences of the indices are the same. Thus we obtain a diagram similar to fig. 50 (iron-filing diagram, fig. 51).

In the middle between two parallel rectilinear conductors, traversed in opposite directions by currents each of strength i deca-ampères, the field intensity is $2.2i \div a/2 = 8i/a$, where a is the distance between the conductors. If a conductor carrying the same current is bent into the form of a circle of diameter a, more lines of force are brought into the plane of the diagram from above and below. By § 165 the field-intensity at the centre of the circle will be $4\pi i/a$ when the current-strength is i. We must accordingly construct our diagram for a current-strength $i\pi/2$ deca-ampères to obtain the nearest approximation to the distribution of lines of force through a circular loop in which the current is i. Thus our diagram will furnish a fair representation of the flux of force through the circuit of a tangent galvanometer traversed by a current i, at least in the more central parts of the field.

Let the two poles, $s = -8/4\pi$ and $n = +8/4\pi$ units, be placed upon the line AA′, symmetrically with respect to its middle point, so that the flux of force proceeding from one pole or entering the other is equal to 8 units. The resultant field is obtained by combining the eight straight lines from the poles s and n with the lines of force already mapped for the biaxal field, due account being taken of the sense in which the lines run. It will be seen that eight lines of force terminate at s, while eight others originate at n and pass out amongst those of the biaxal field. Thus we obtain a diagram corresponding to fig. 61. The tension along the lines of force tends to deflect the needle in the sense indicated by the thumb rule.

When on the field already considered we superpose the uniform terrestrial magnetic field in the direction AA′ (§ 91), we have a representation of all the magnetic forces concerned in the working of the tangent galvanometer.

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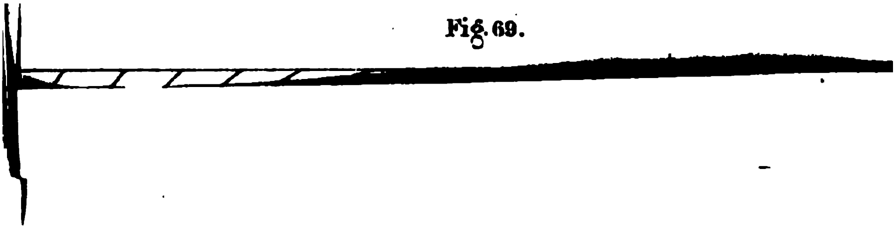
Fig. 69.



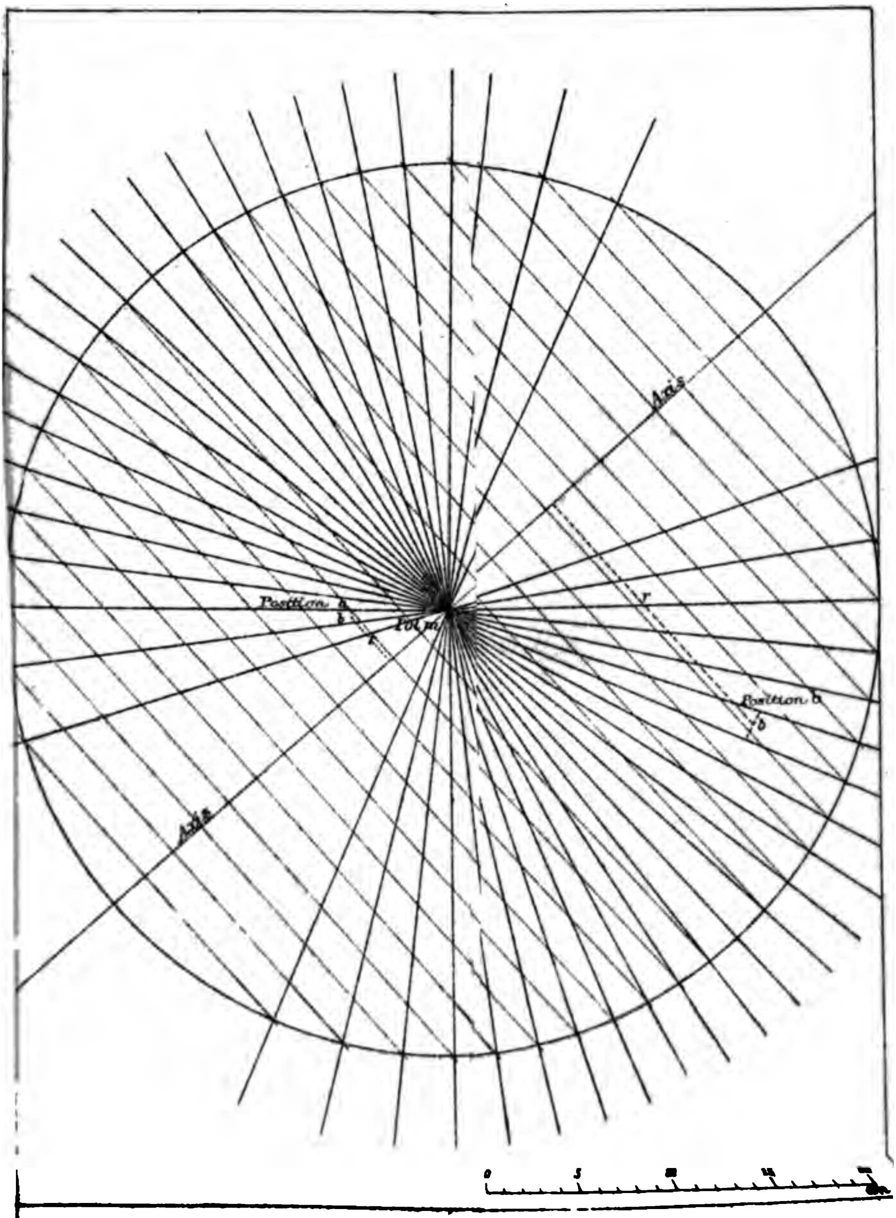


Fig. 89.

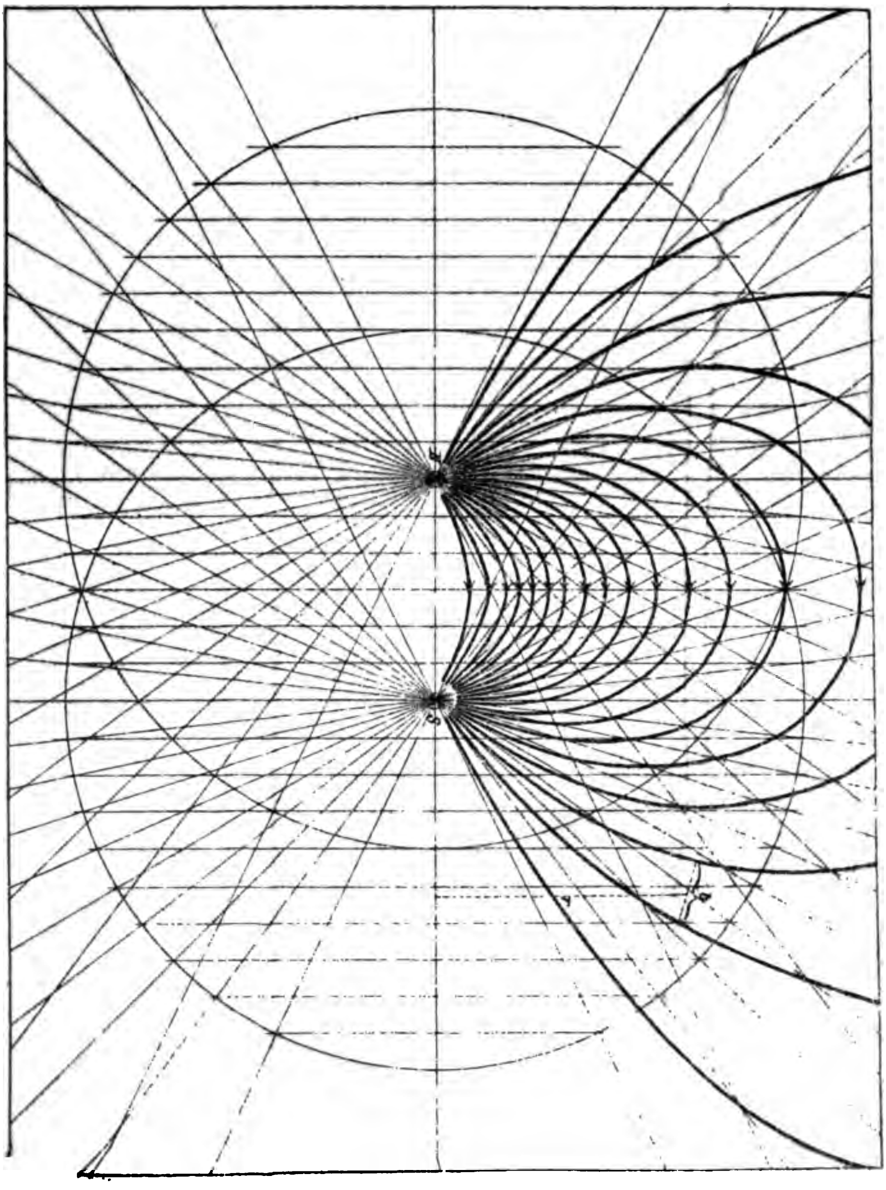


Fig. 26.

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